TESTING GENERAL RELATIVITY WITH GRAVITATIONAL WAVES

A "NO-HAIR" TEST FOR BINARY BLACK HOLES

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on behalf of
the LIGO Scientific Collaboration and Virgo Collaboration

DISCLAIMERS

- testing the no-hair theorem doesn't mean we don't trust the theorem
  - on the contrary the theorem helps to confirm if merger remnants are truly black holes
- we will assume that astrophysical black holes carry no electronic charge
  - LIGO's discoveries are possibly from Kerr black holes
  - in principle we can constrain the magnitude of charge

OVERVIEW

- null tests of general relativity
- tests of propagation of propagation
- test of black hole hypothesis of LIGO's detections

WHY ARE BINARY BLACK HOLES GOOD TESTBEDS FOR GENERAL RELATIVITY?

$$G_{\mu\nu} = \frac{1}{\kappa} T_{\mu\nu}, \quad \kappa = \frac{c^4}{8\pi G} \sim 5 \times 10^{49}\,\mathrm{dynes}$$

$$L \sim \frac{32\eta^2 c^5}{5G} \left(\frac{v}{c}\right)^{10}, \quad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}, \quad \frac{2c^5}{5G} \simeq 10^{59}\,\mathrm{erg\,s^{-1}}$$

TESTS OF GR AND ALTERNATIVES

- null tests of GR
  - assume that GR is correct and look for small deviations from GR
    - e.g. search for tails of gravitational waves
- tests of alternative theories of gravity
  - modified phase evolution of GR waveform by introducing new terms
  - could potentially arise from an alternative theory
    - e.g. massive graviton, dipole radiation, scalar modes, …

NULL TESTS WITH LIGO'S DETECTIONS

$$\tilde{h}(f) = \mathcal{A}(f) e^{i\varphi(f)}$$

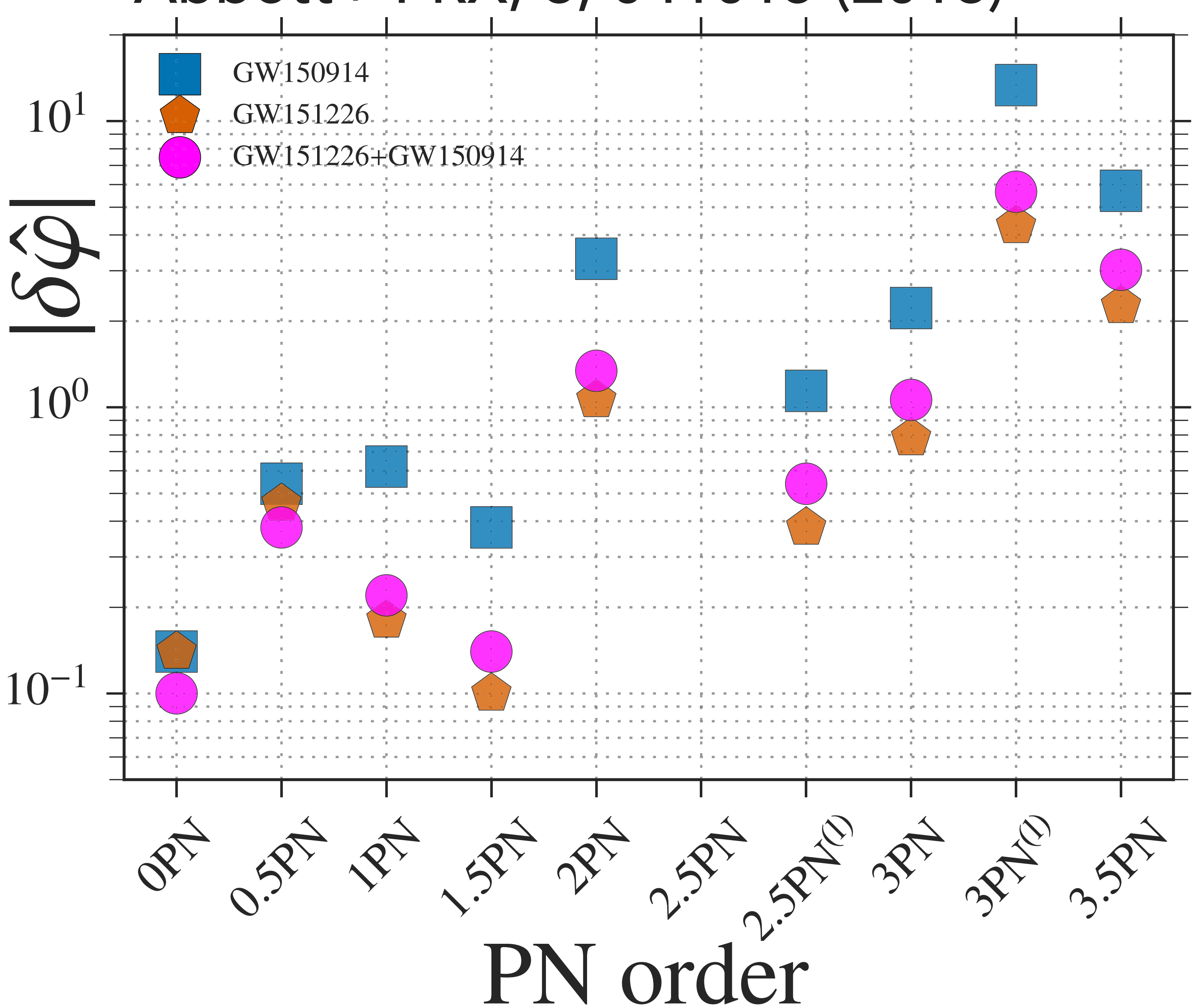
$$\varphi(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + \varphi_{\rm Newt}(Mf)^{-5/3} + \varphi_{\rm 0.5PN}(Mf)^{-4/3} + \varphi_{\rm 1PN}(Mf)^{-1} + \varphi_{\rm 1.5PN}(Mf)^{-2/3} + \cdots$$

Blanchet and BS 1995

Arun+ 2006, Mishra+ 2010,

Yunes and Pretorius 2009, Li+ 2012

Abbott+ PRX, 6, 041015 (2016)



wave tails

mass asymmetry

spin-spin coupling

spin-orbit coupling

hereditary terms

precession

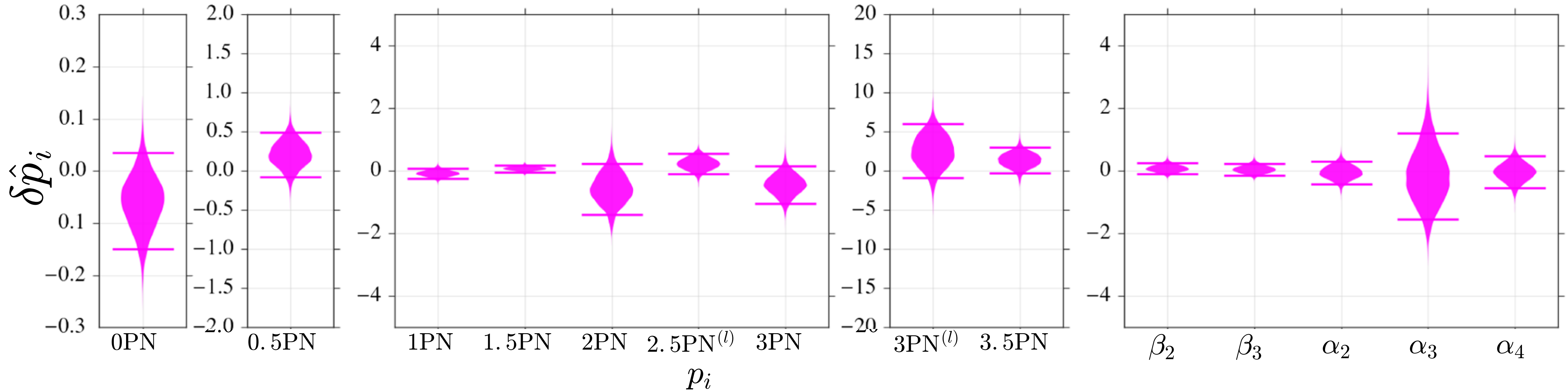
absorption of radiation by black hole

deform PN coefficients from their GR value and look for them in the data

$$\varphi_{\rm Newt} \to \varphi_{\rm Newt} + \delta\varphi_{\rm Newt}$$

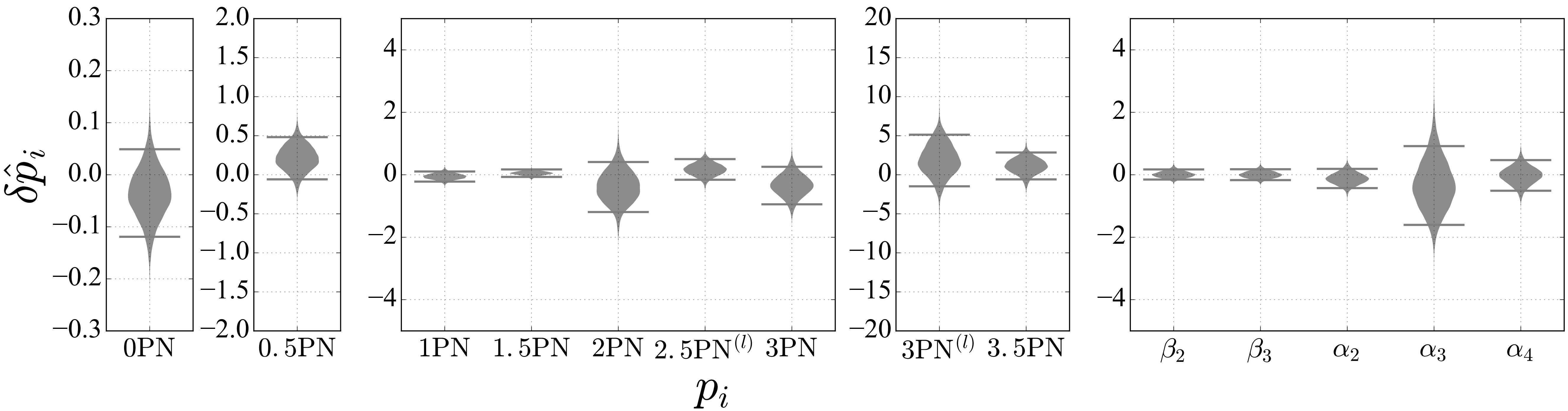
TESTS USING IMR WAVEFORM MODEL

GW150914+GW151226



Abbott+ PRX, 6, 041015 2016

GW170104



Abbott+ PRL, 118, 221101 (2017)

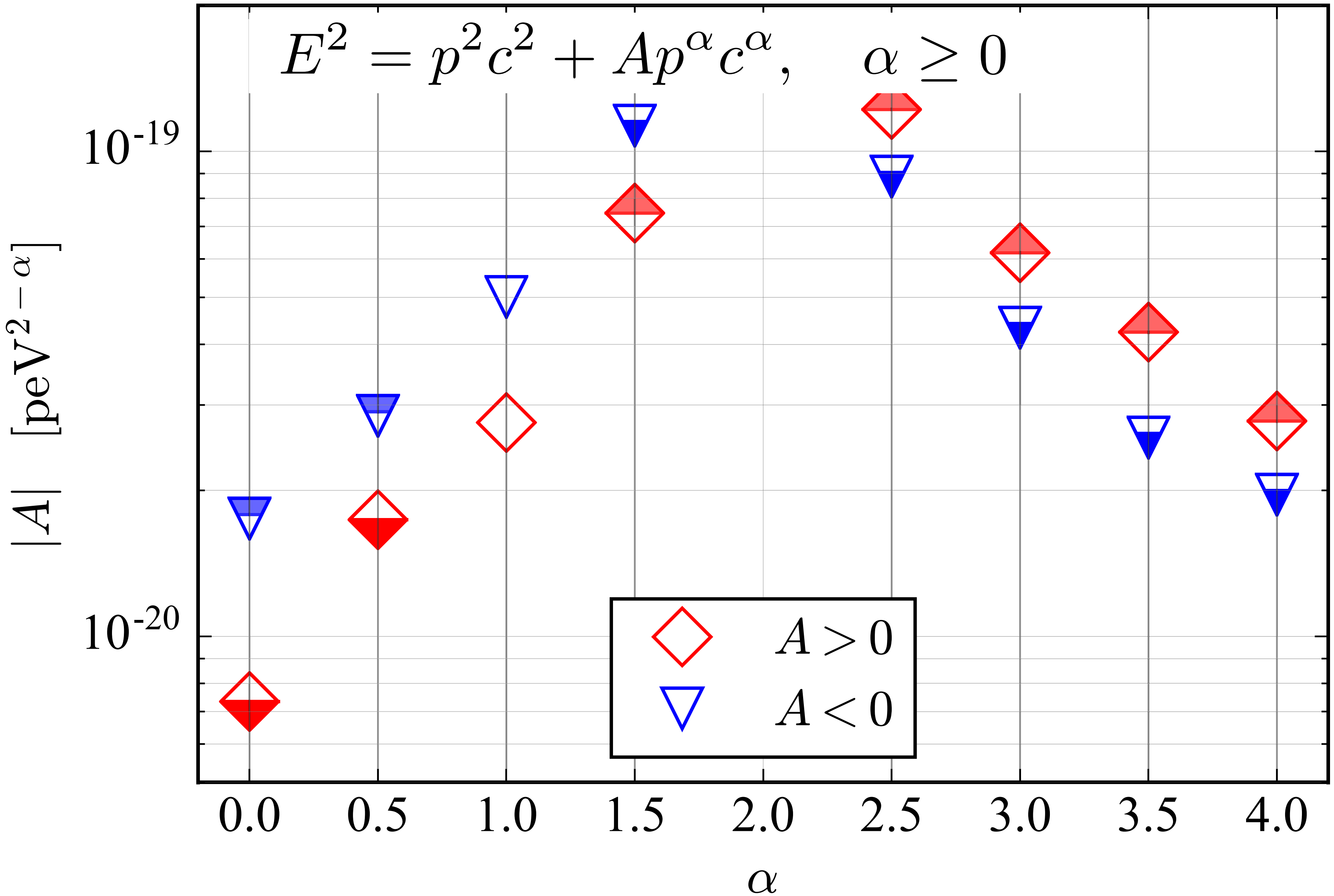
MODIFIED DISPERSION RELATIONS

$$E^2 = p^2c^2 + Ap^\alpha c^\alpha, \quad \alpha \geq 0$$

- many theories of gravity predict gravitational waves could be dispersed
- this is the first time we have tested for generic dispersion relation for gravitational waves
  - first constraints in the gravity sector for superluminal gravitational waves
  - find no evidence for dispersion of gravitational waves
  - **GW170104** bound on graviton mass: $< 7.7 \times 10^{-23}/c^2$

Abbott+ PRL, 118, 221101 (2017)

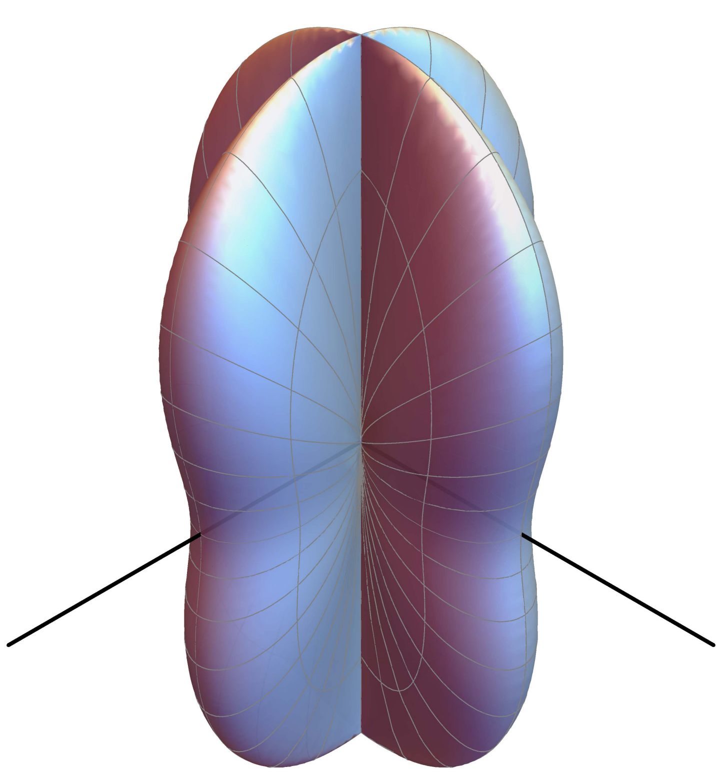
CONSTRAINTS ON DISPERSION OF GRAVITATIONAL WAVES



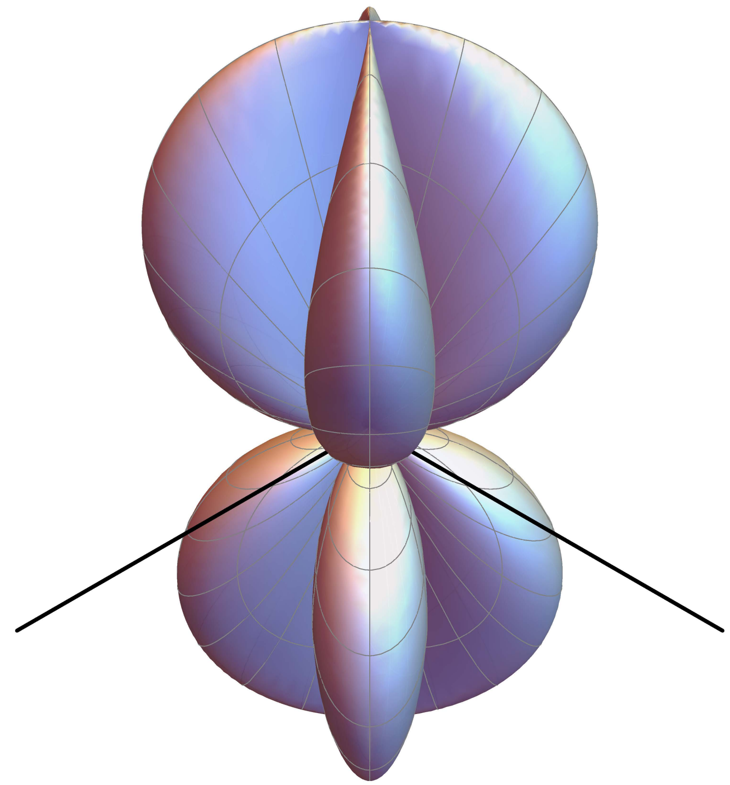
massive-graviton theories (α=0, $A > 0$), multifractal spacetime (α=2.5), doubly special relativity (α=3), and Hořava-Lifshitz and extra-dimensional theories (α=4).

Abbott+ PRL, 118, 221101 (2017)

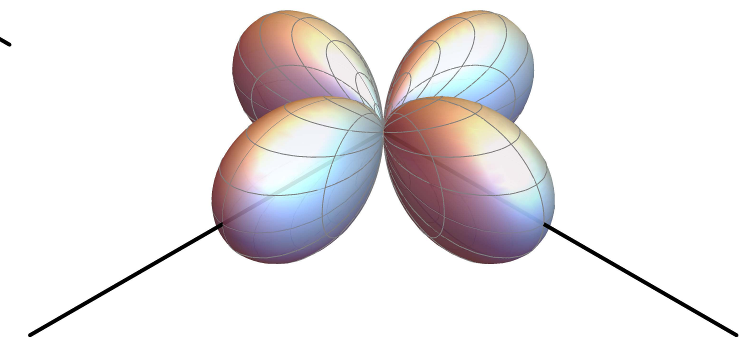
ANTENNA PATTERN FUNCTIONS DUE TO TENSOR, VECTOR AND SCALAR POLARIZATIONS



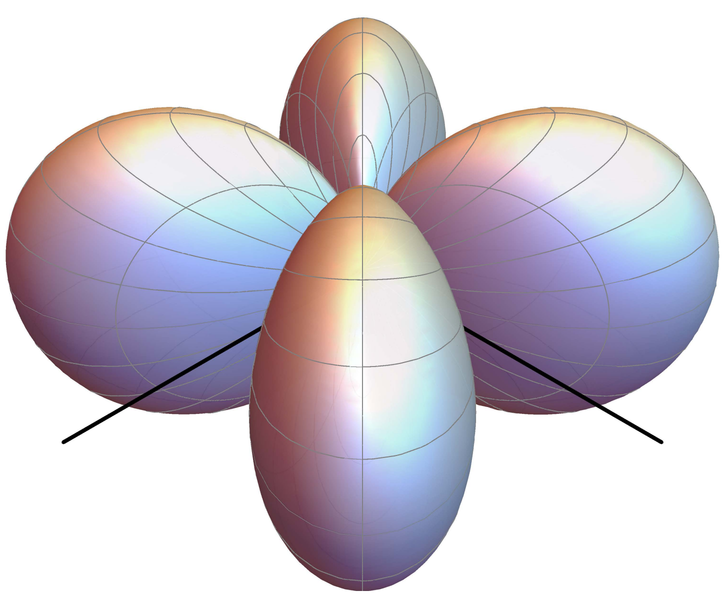
Tensor + mode



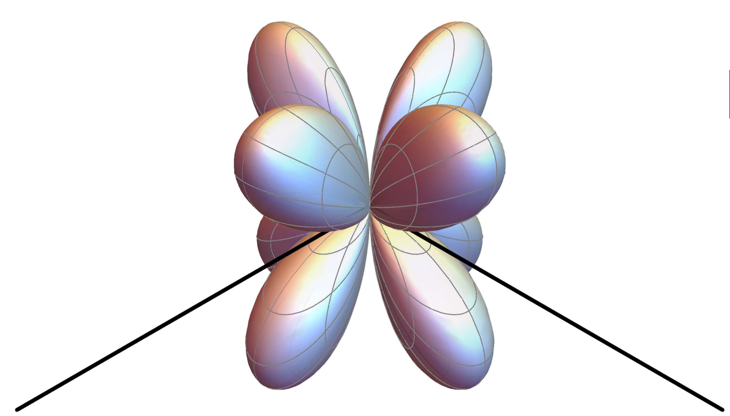
Tensor x mode



scalar mode
longitudinal and transverse



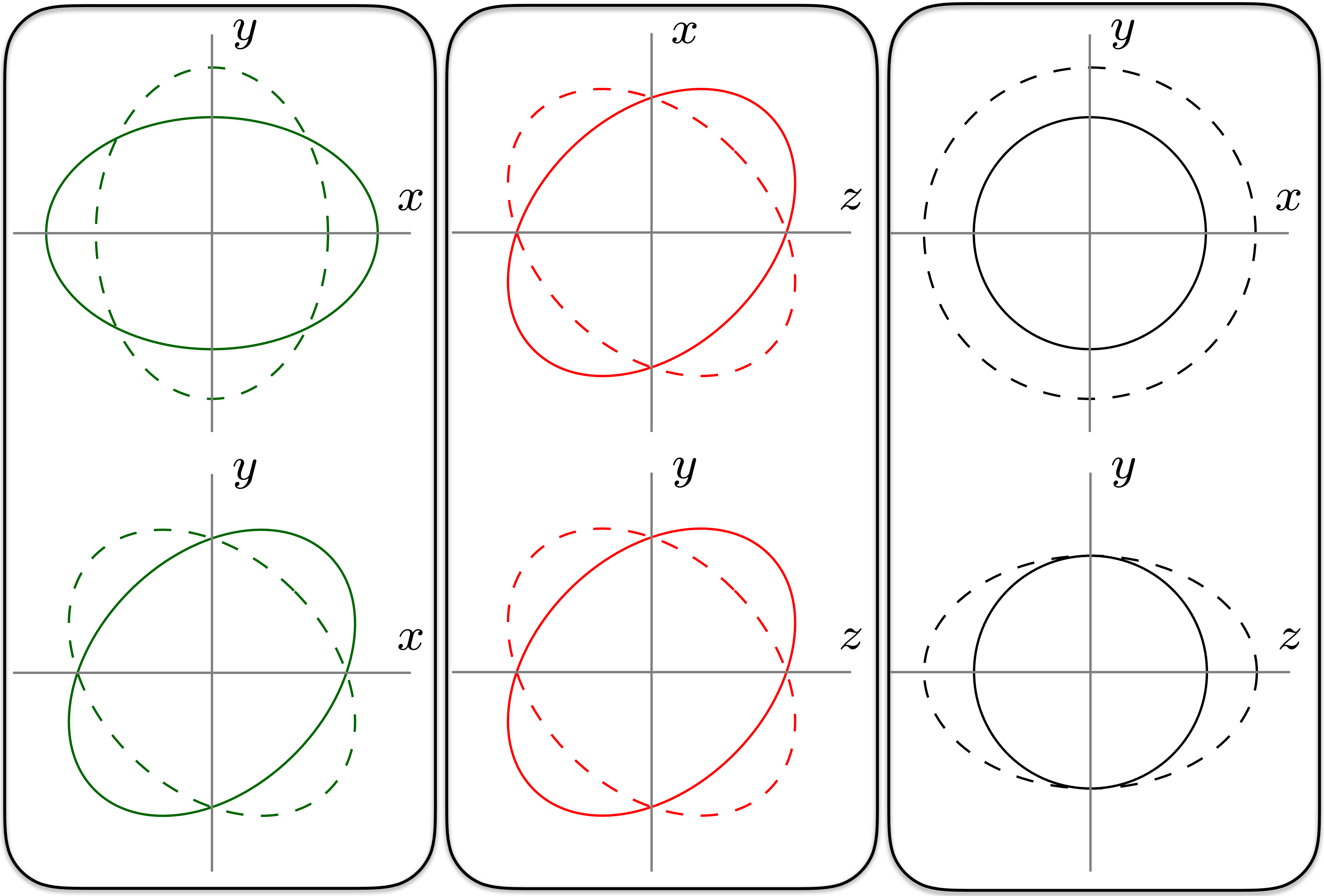
Vector x mode



Vector y mode

LVC/EPO, Max Isi

DEFORMATION OF A RING OF FREE PARTICLES BY DIFFERENT POLARIZATIONS



Isi+, Phys Rev D. 96 042001

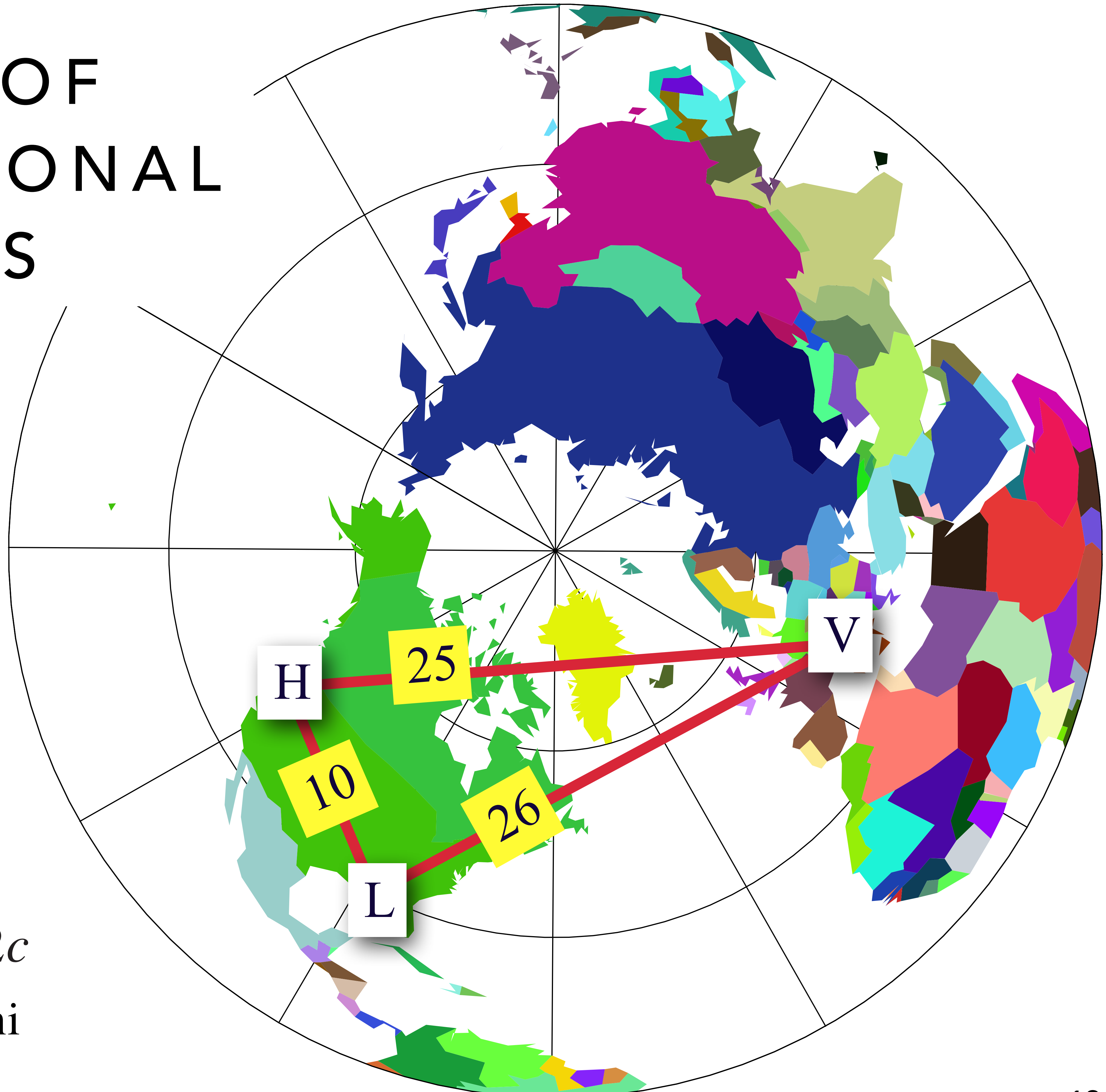
TENSOR VS SCALAR/VECTOR POLARIZATION: GW170814

- computed the Bayes factor for purely tensor modes vs purely scalar modes AND purely tensor modes vs purely vector modes
- compared to purely vector mode purely tensor mode is favored by a Bayes factor of 1:200
- compared to purely vector mode purely tensor mode is favored by a Bayes factor of 1:1,000
- 3 detectors have very poor sensitivity to a mixture model of tensor-vector-scalar modes

Phys Rev Lett. 119, 141101 (2017)

SPEED OF GRAVITATIONAL WAVES

BASELINES IN LIGHT TRAVEL TIME (MS)



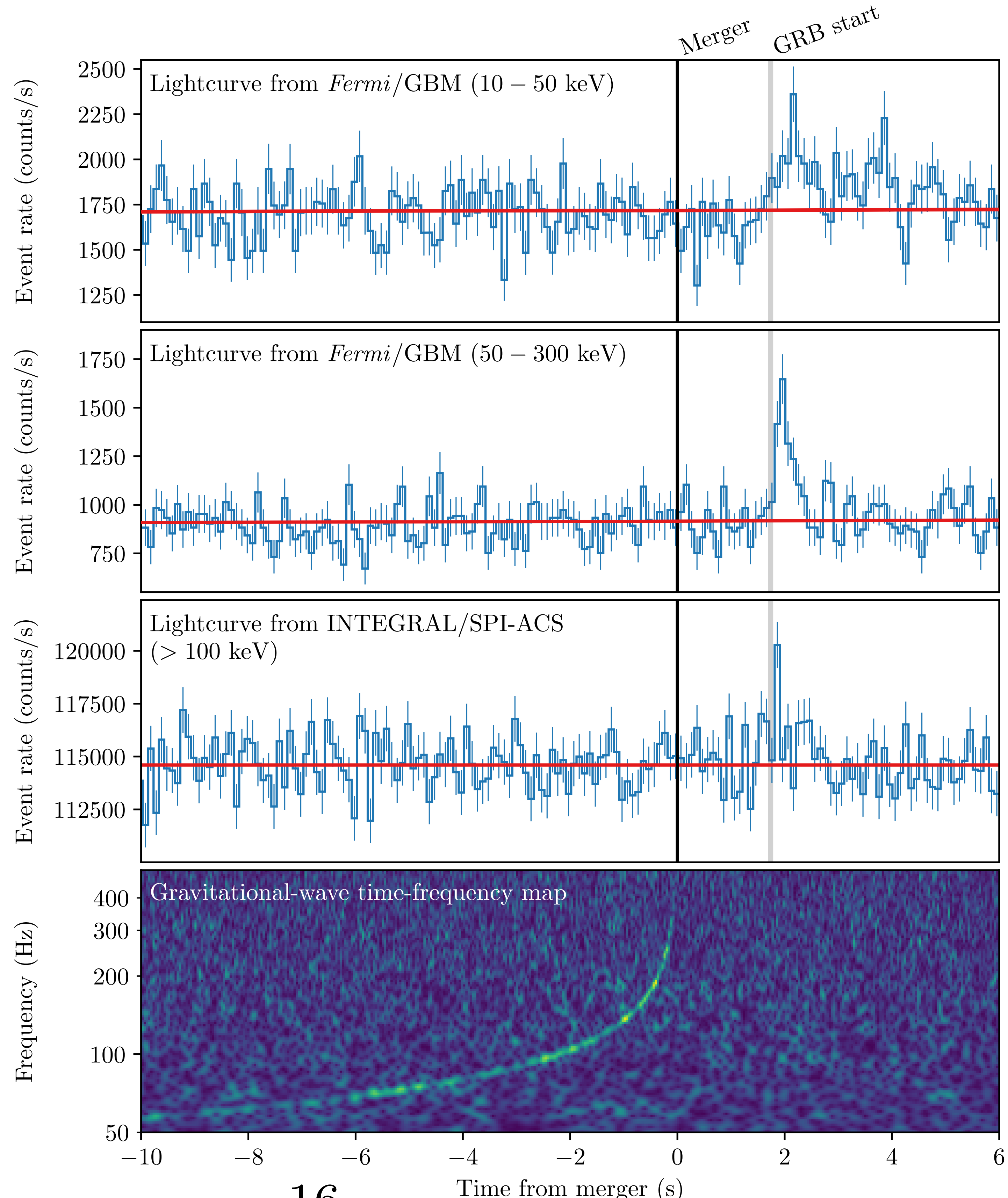
USING GW150914, GW151226, GW170104

$$0.55c < c_{\rm gw} < 1.42c$$

Cornish, Blas Nardini

PRL **119**, 161102 (2017)

SPEED OF GRAVITATIONAL WAVES

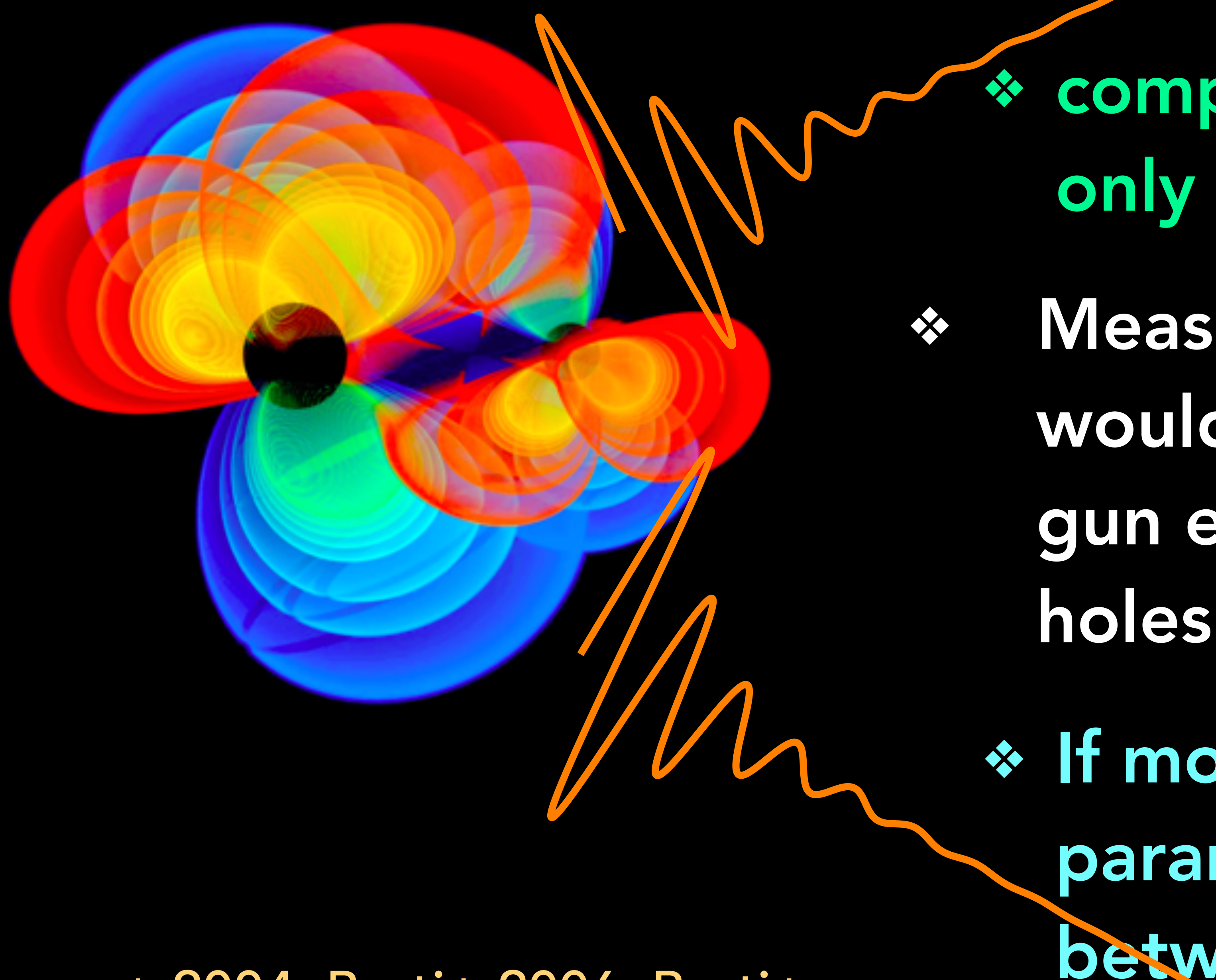


$$\frac{1.7\,\mathrm{s}}{40\,\mathrm{Mpc}/c} \sim 4 \times 10^{-16}$$

$$-3 \times 10^{-15} \leq \frac{v_{\mathrm{GW}} - v_{\mathrm{EM}}}{v_{\mathrm{EM}}} \leq 7 \times 10^{-16}$$

THE ASTROPHYSICAL JOURNAL LETTERS, 848:L13 (27pp), 2017

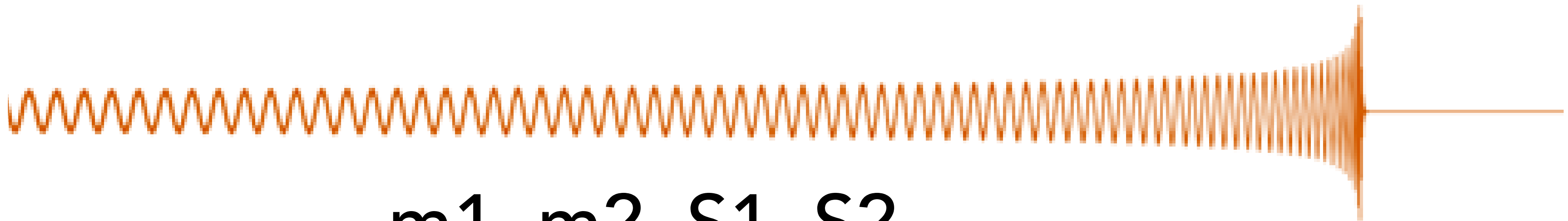
QUASI-NORMAL MODES AND NO-HAIR



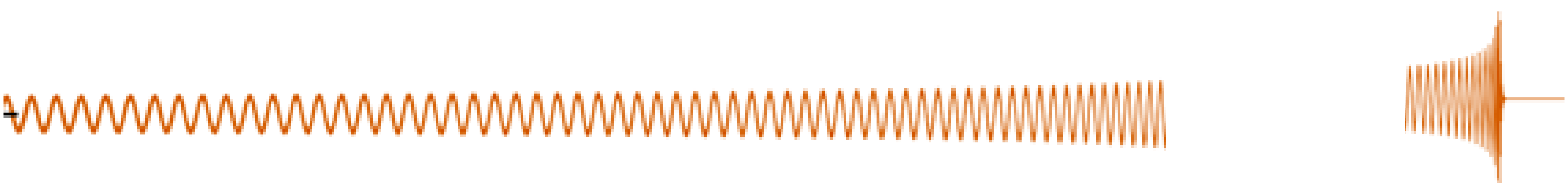
- Deformed black holes emit quasi-normal modes
  - complex frequencies depend only on the mass and spin
- Measuring two or modes would provide a smoking gun evidence of Kerr black holes
  - If modes depend on other parameters, consistency between different mode frequencies would fail

Dreyer+ 2004, Berti+ 2006, Berti+ 2007, Kamaretsos+ 2012, Gossan +2012, Bhagwat+ 2017, Brito+ 2018

IS THE SIGNAL CONSISTENT WITH A BLACK HOLE REMNANT



m1, m2, S1, S2, …



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Abbott+ PRL, 118, 221101 (2017)

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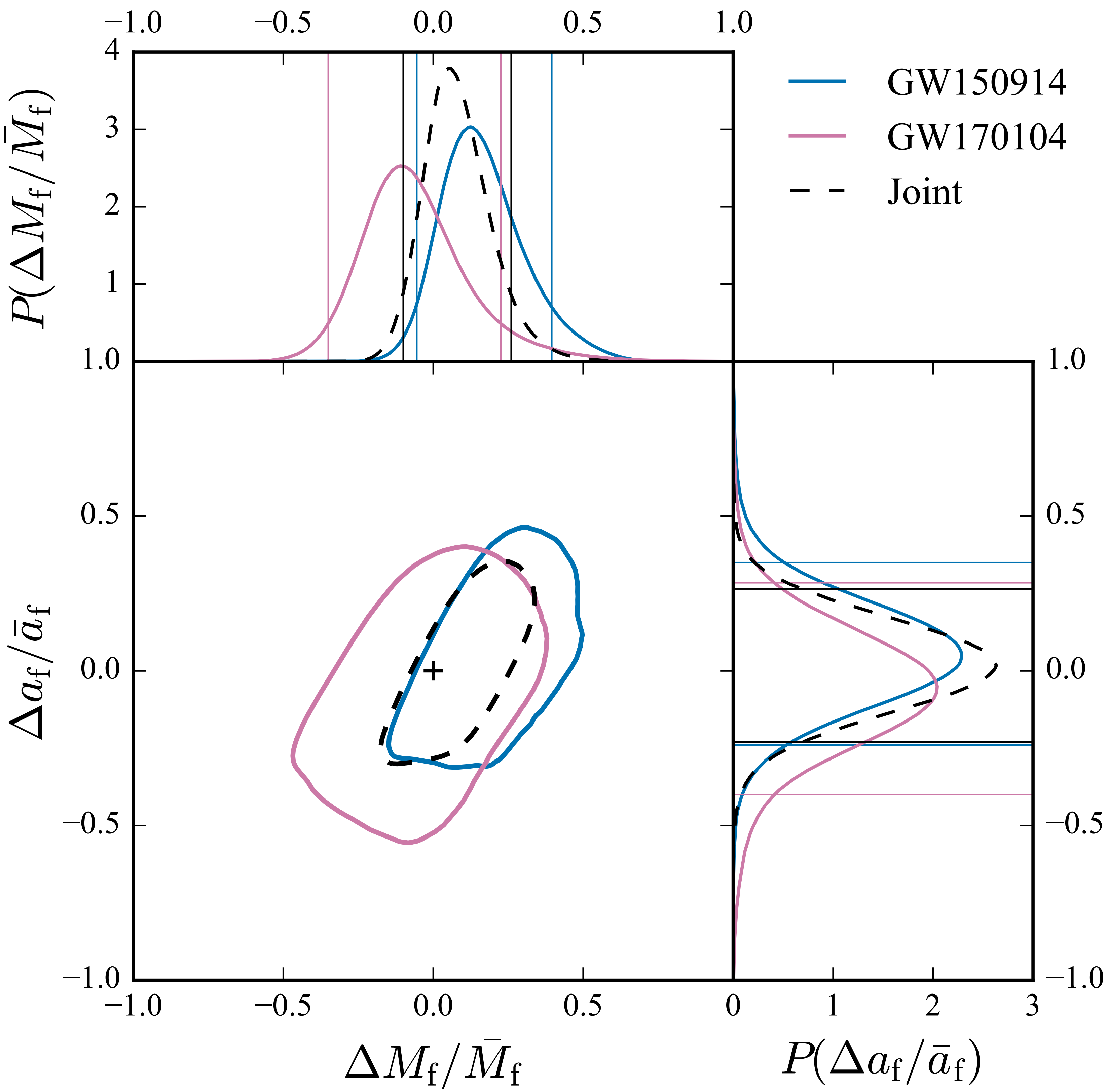
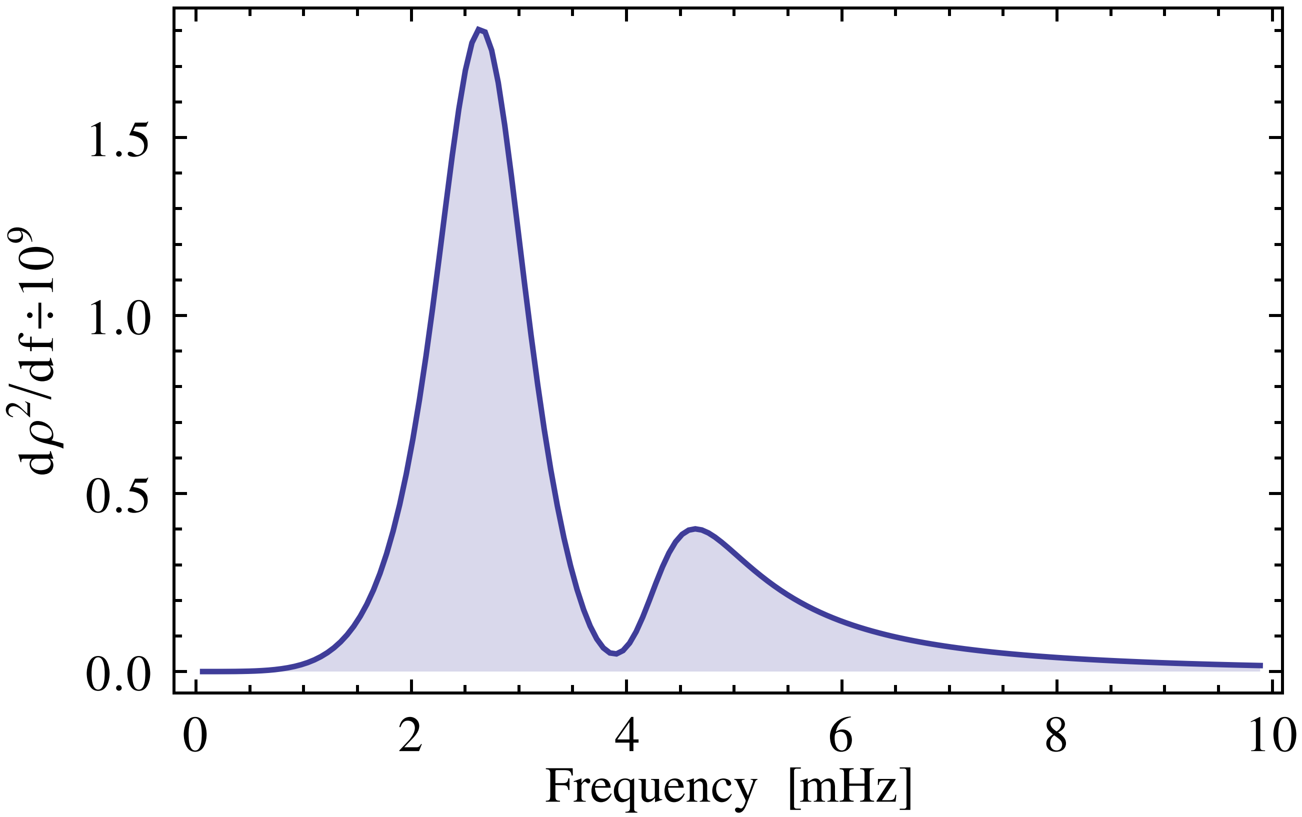
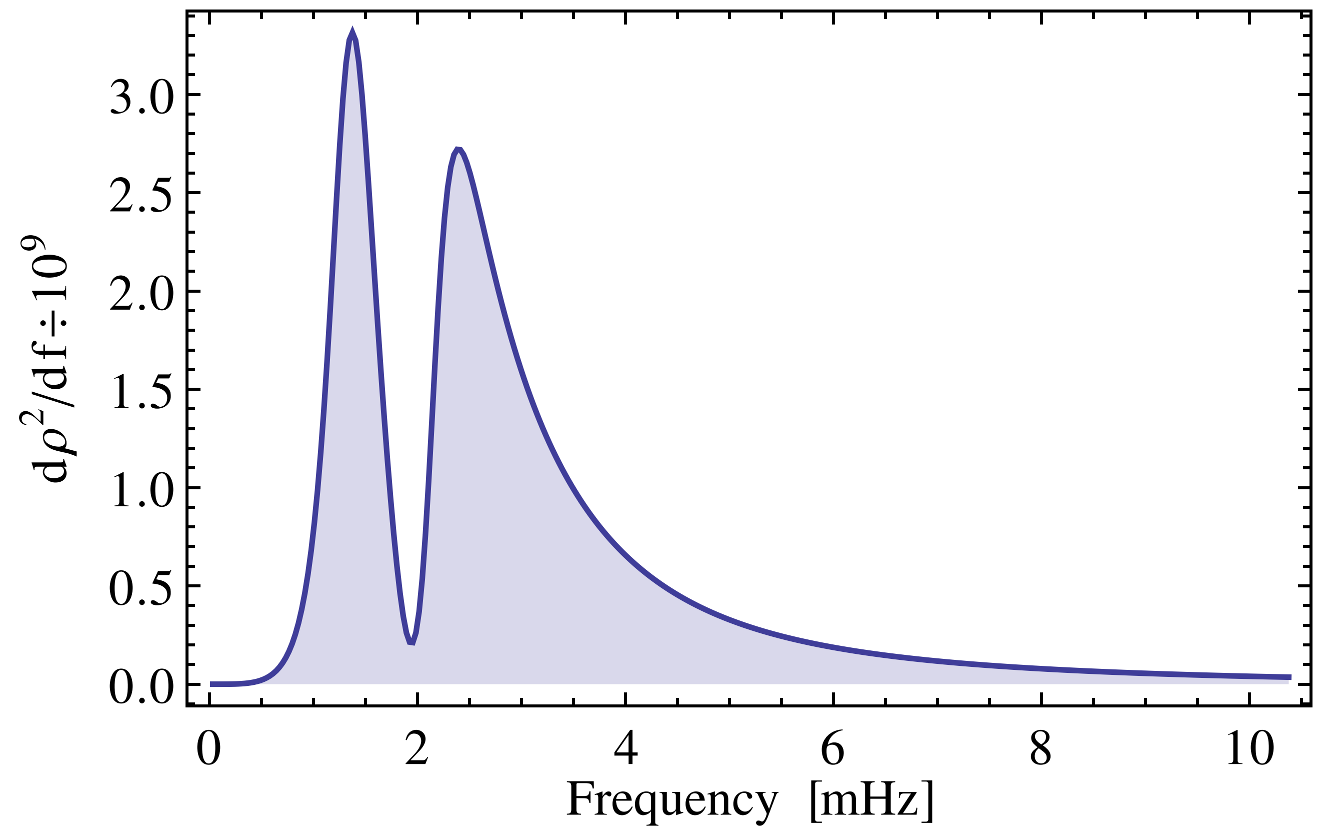


Abbott+ PRL, 118, 221101 (2017)

QNM SIGNATURE IN BINARY BLACK HOLES

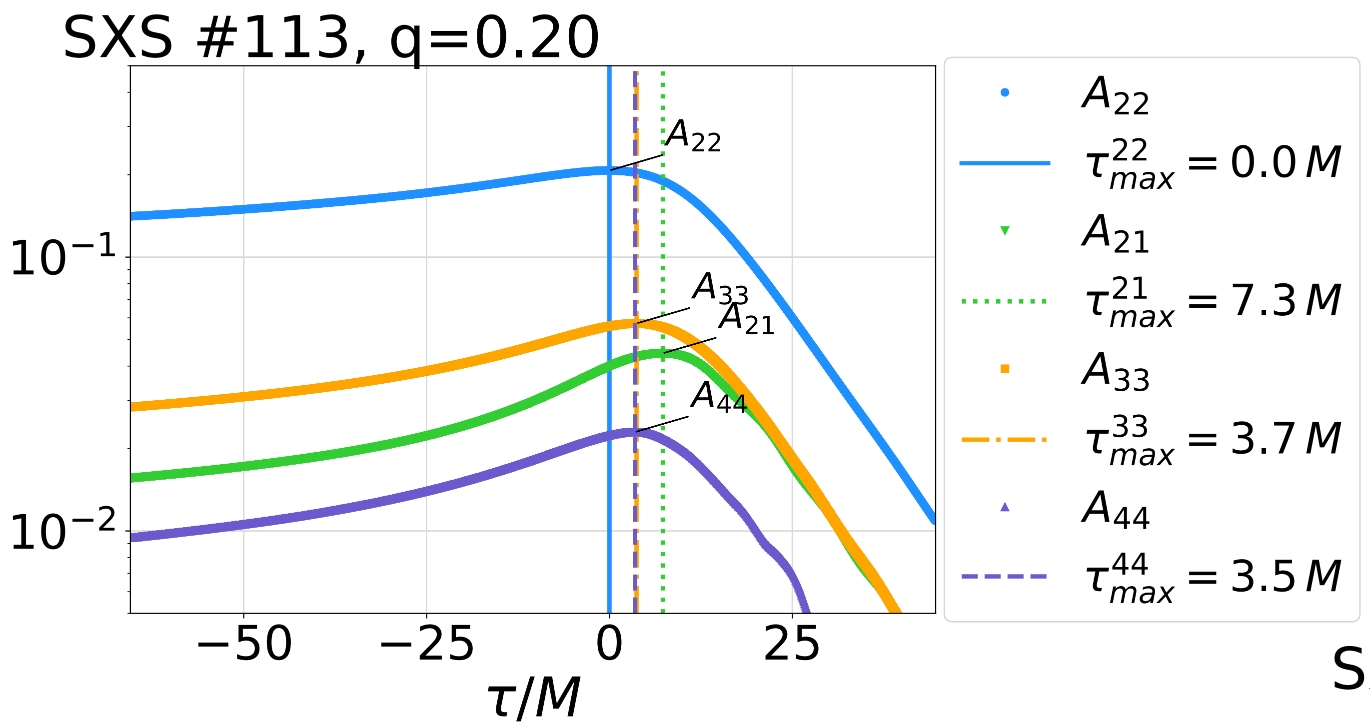
- Quasi normal mode signal example in LISA:
  - left total mass of 5 million solar mass
  - right total mass of 50 million solar masses
- mass ratio of 1:10 in both cases

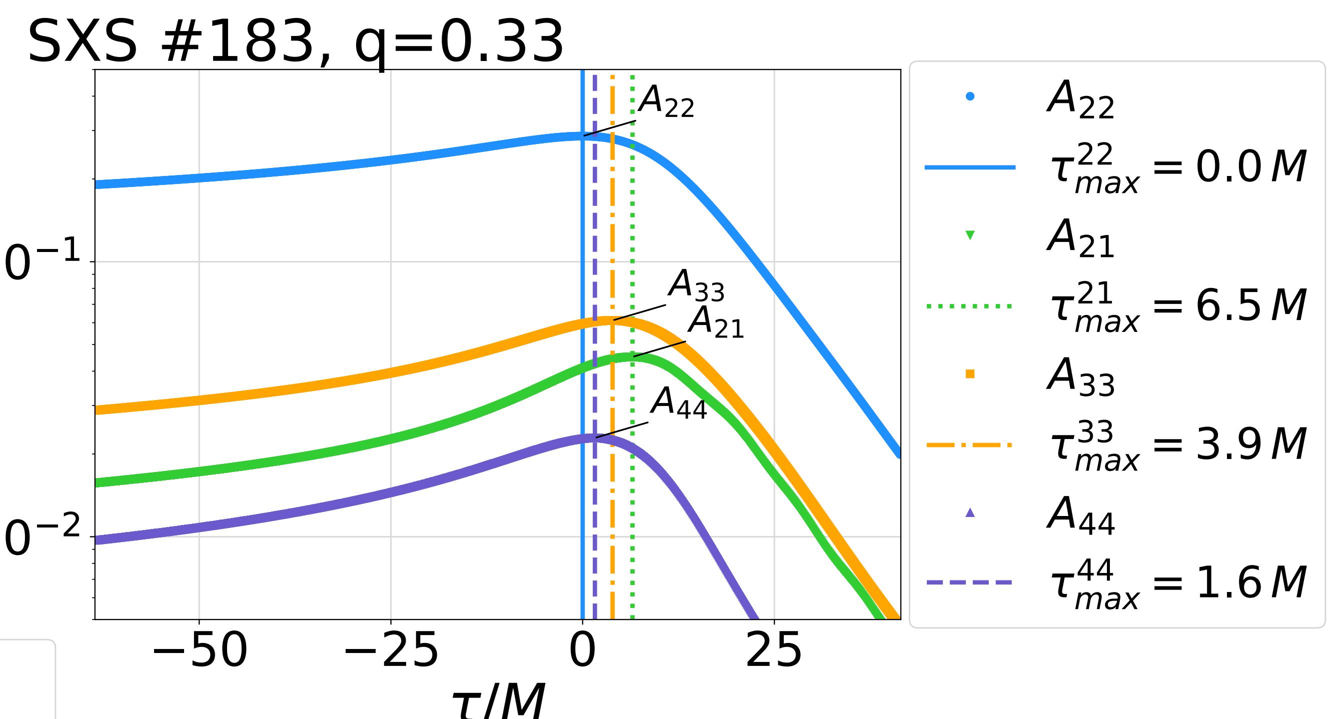


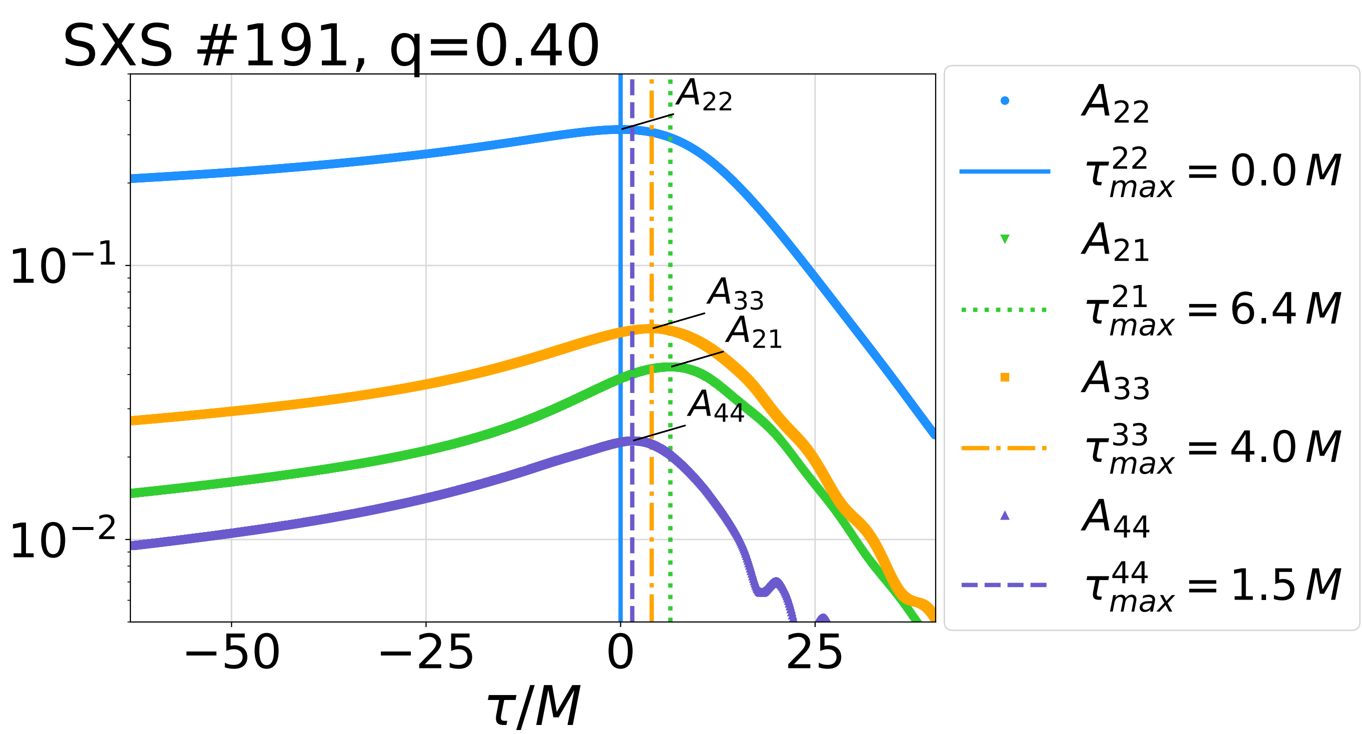


Kamaretsos+ 2012

HIGHER MODES FROM INSPIRAL TO RINGDOWN

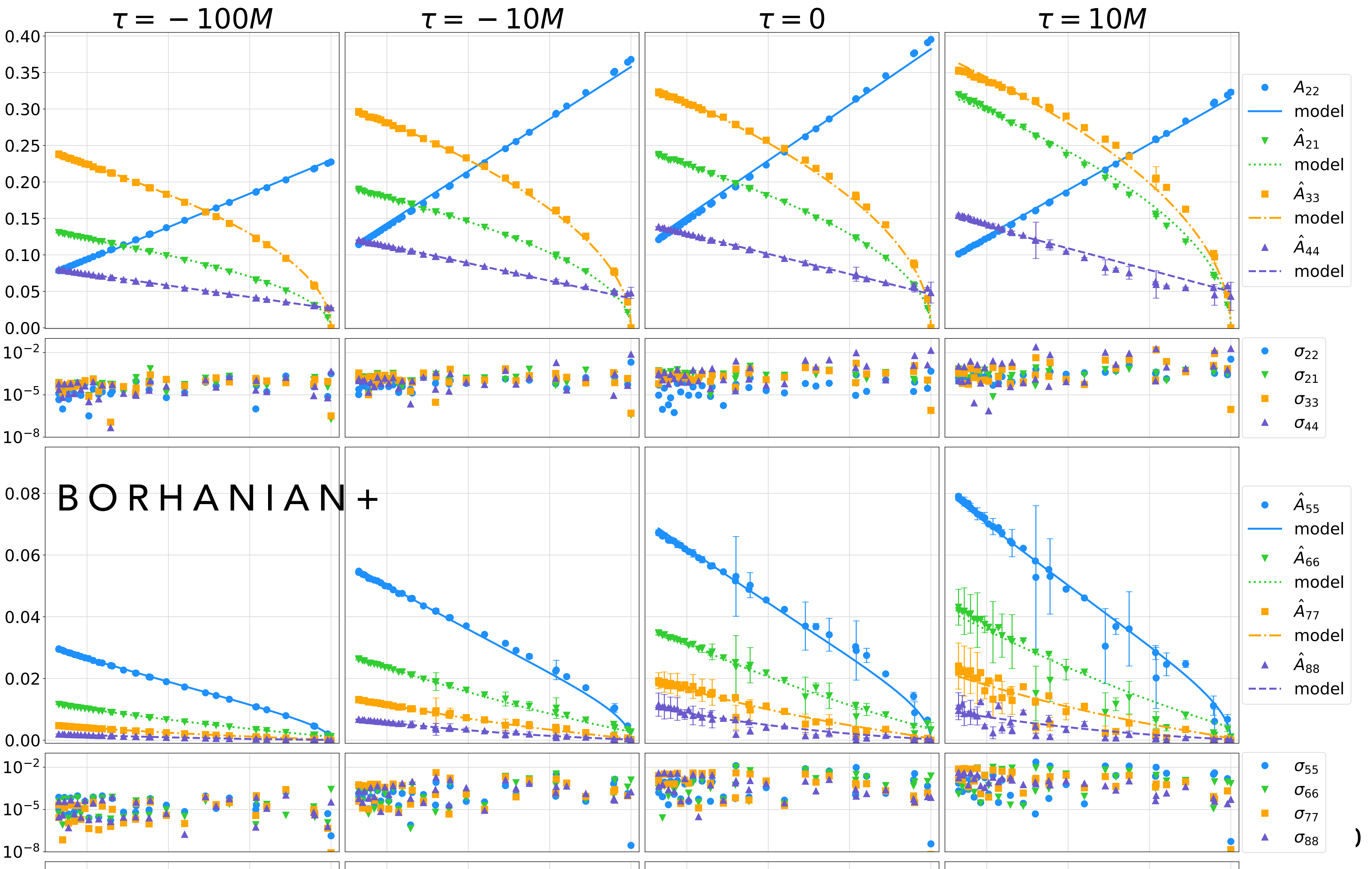




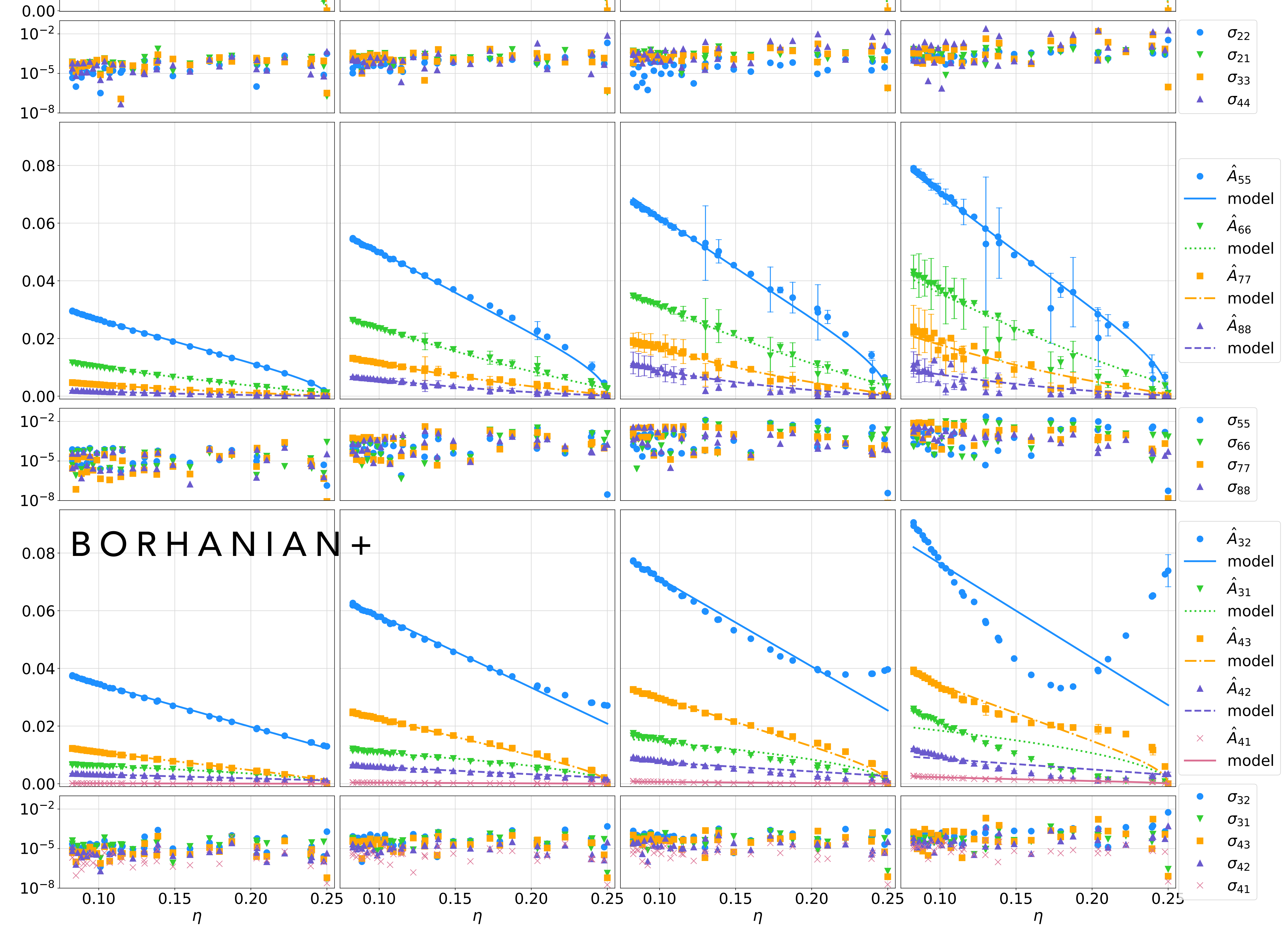


BORHANIAN+

SIGNATURE OF NONLINEARITY IN BINARY BLACK HOLE MERGERS



BORHANIAN+


BORHANIAN+
σ_{22}
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σ_{33}
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$\hat{A}_{55}$
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model
$\hat{A}_{88}$
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$\hat{A}_{42}$
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σ_{32}
σ_{31}
σ_{43}
σ_{42}
σ_{41}
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0.02
0.04
0.06
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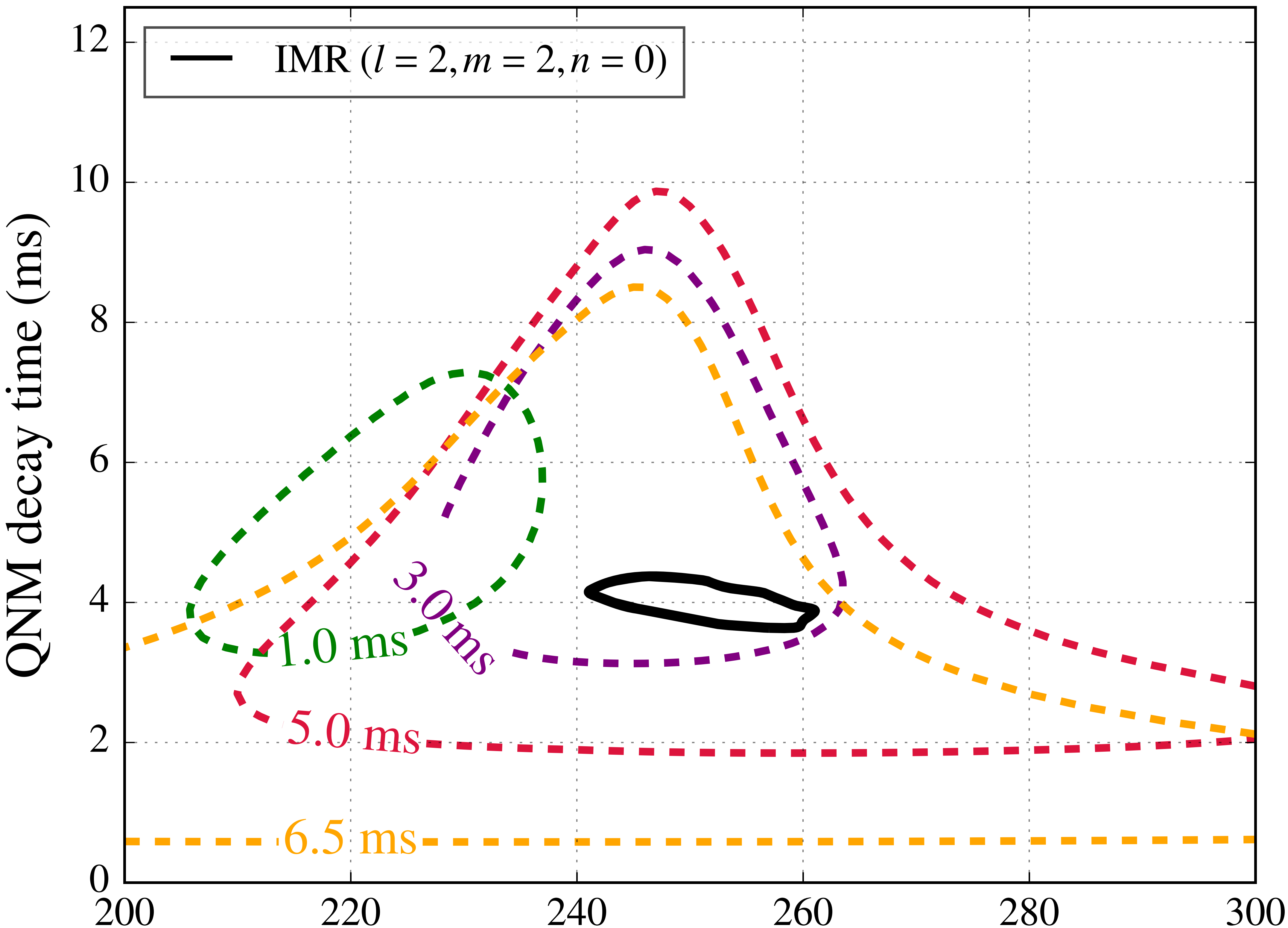
10^{-2}
10^{-5}
10^{-8}
0.10
0.15
0.20
0.25
η

QUASI-NORMAL MODES IN LIGO'S BLACK HOLES? GW150914



PRL **116,** 061102 (2016)

HOW MIGHT ONE TEST THE NO HAIR THEOREM?

- in general relativity the parameters of QNM signal are

$$\vec{\theta}_{\rm GR} = \{M, \nu, j, \chi_{\rm eff}, D_{\rm L}, \theta, \varphi, \psi, \iota, \phi, t_0\}\,,$$

- extra hair:

$$\begin{aligned}\omega_{lm} &= \omega_{lm}^{\rm GR}(M,J)\,(1+\delta\hat{\omega}_{lm}),\\ \tau_{lm} &= \tau_{lm}^{\rm GR}(M,J)\,(1+\delta\hat{\tau}_{lm}),\end{aligned}$$

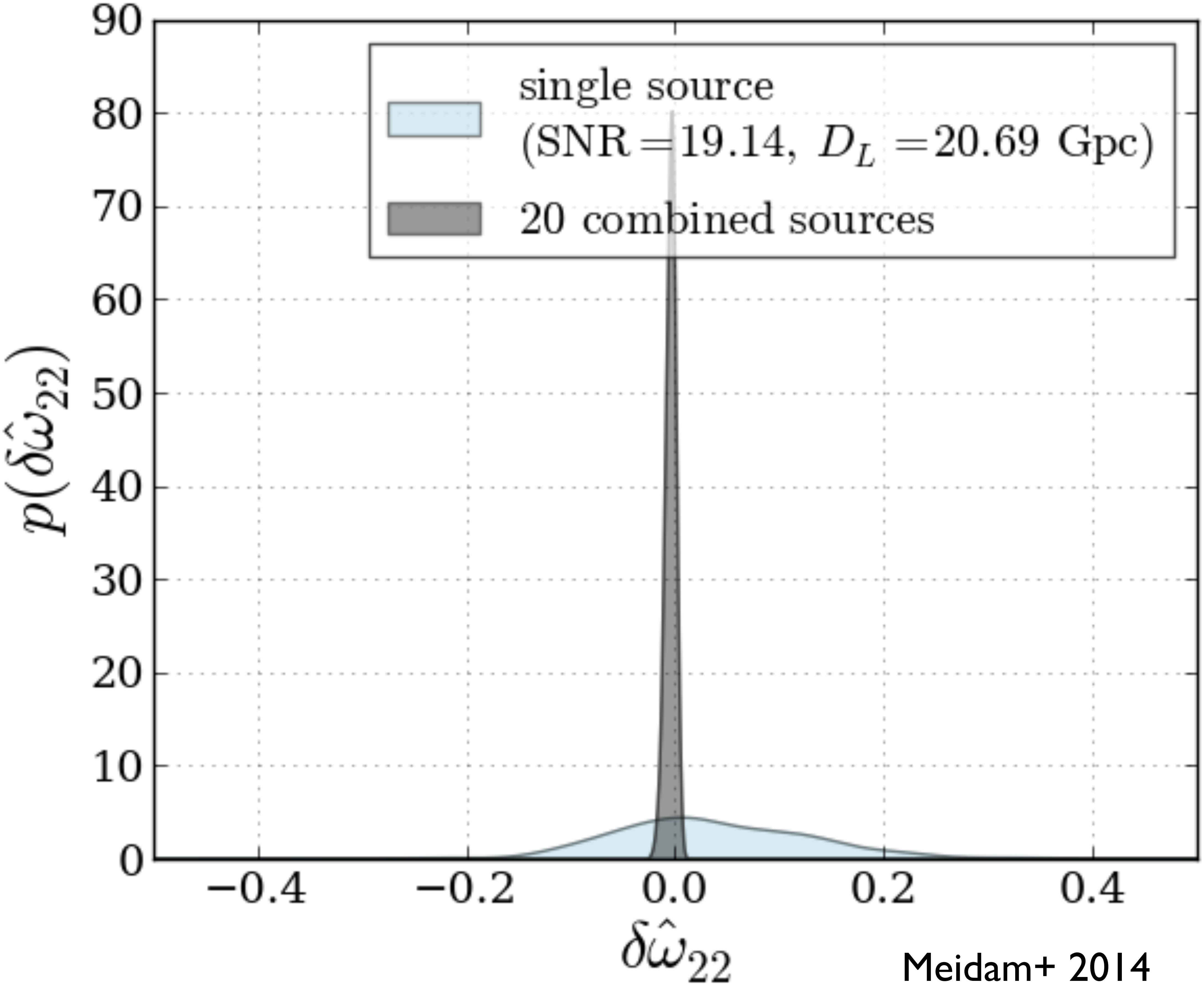
$$\begin{aligned}H_1 &\longleftrightarrow \{\vec{\theta}_{\rm GR}, \delta\hat{\omega}_{22}\},\\ H_2 &\longleftrightarrow \{\vec{\theta}_{\rm GR}, \delta\hat{\omega}_{33}\},\\ H_3 &\longleftrightarrow \{\vec{\theta}_{\rm GR}, \delta\hat{\tau}_{22}\},\\ H_{12} &\longleftrightarrow \{\vec{\theta}_{\rm GR}, \delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}\},\end{aligned}$$

$\Rightarrow$

$$B^{123}_{\rm GR} = \frac{P(d|H_{123}, I)}{P(d|\mathcal{H}_{\rm GR}, I)}.$$

Gossan+ 2012, Meidam+ 2014

HOW WELL CAN WE MEASURE NON-GR



Meidam+ 2014

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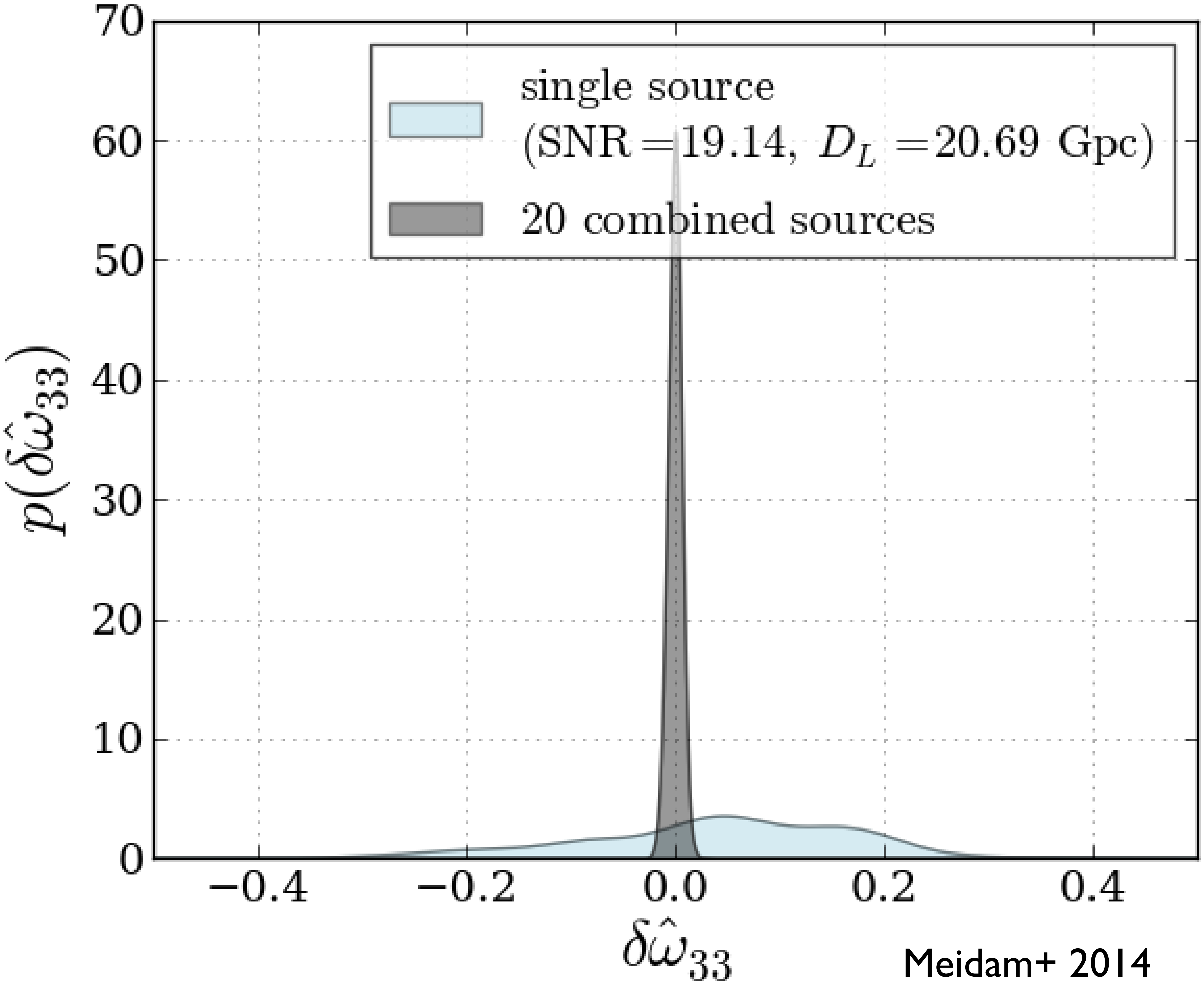
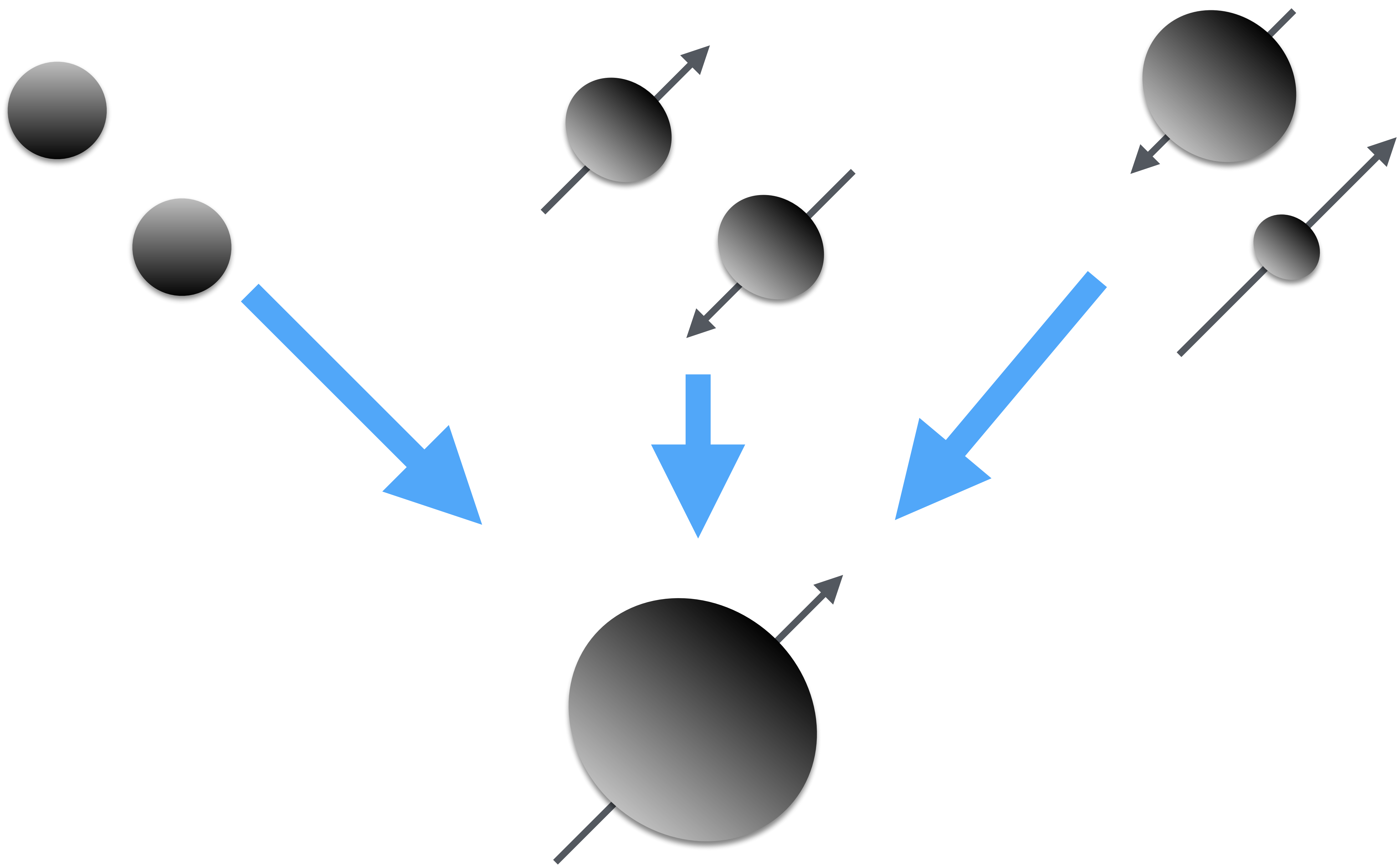


Meidam+ 2014

NOT ALL BLACK HOLES ARE BORN EQUAL

- infinitely many binaries can form the same final black hole



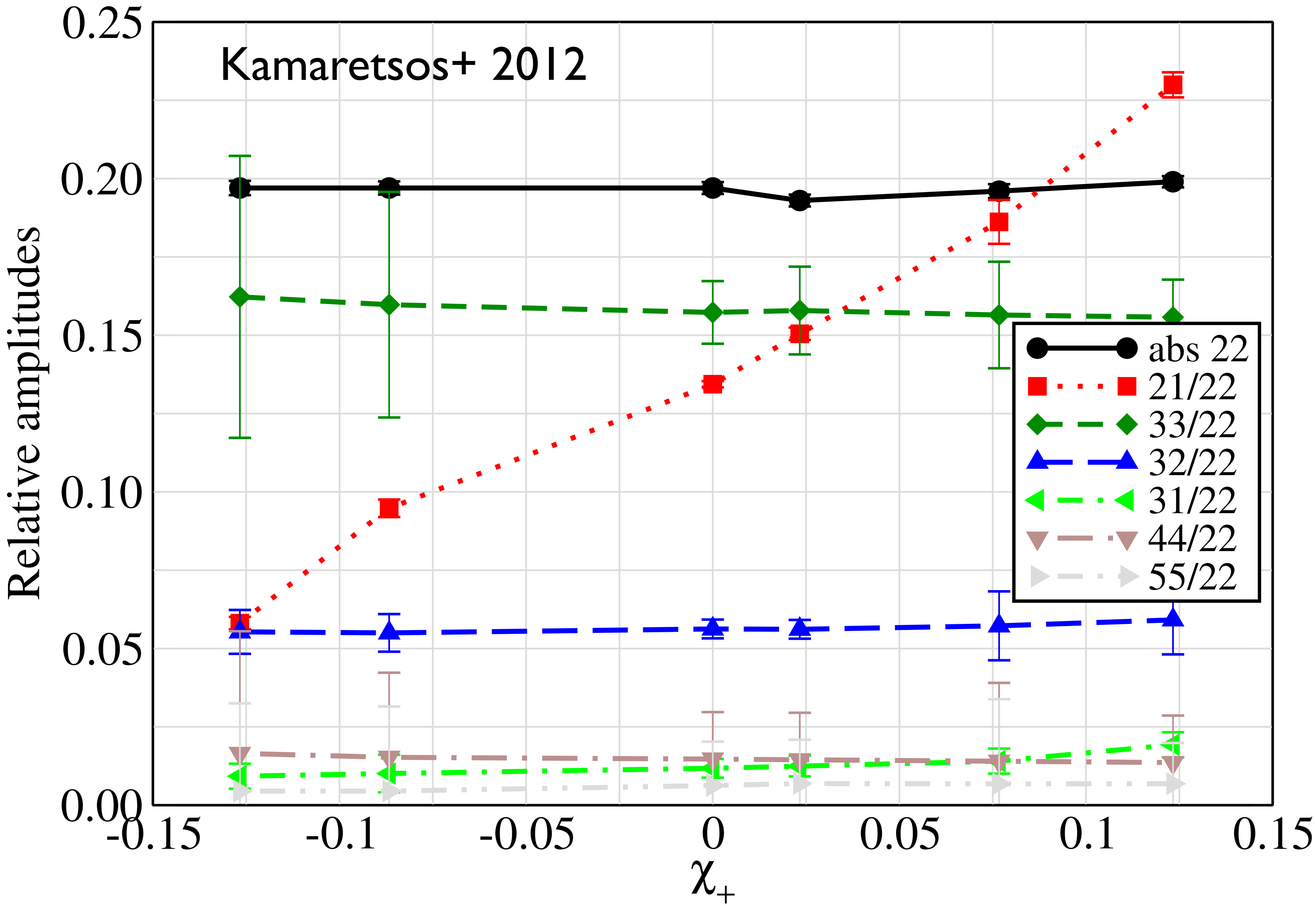
BLACK HOLES NEVER FORGET

22 AND 33 MODE AMPLITUDES ARE CONSTANT

21 MODE VARIES WITH TOTAL SPIN

$$\chi_+ = (m_1\,\chi_1 + m_2\,\chi_2)/M_{in}$$

Initial spins such that χ ~ 0.62, mass ratio *q*=2



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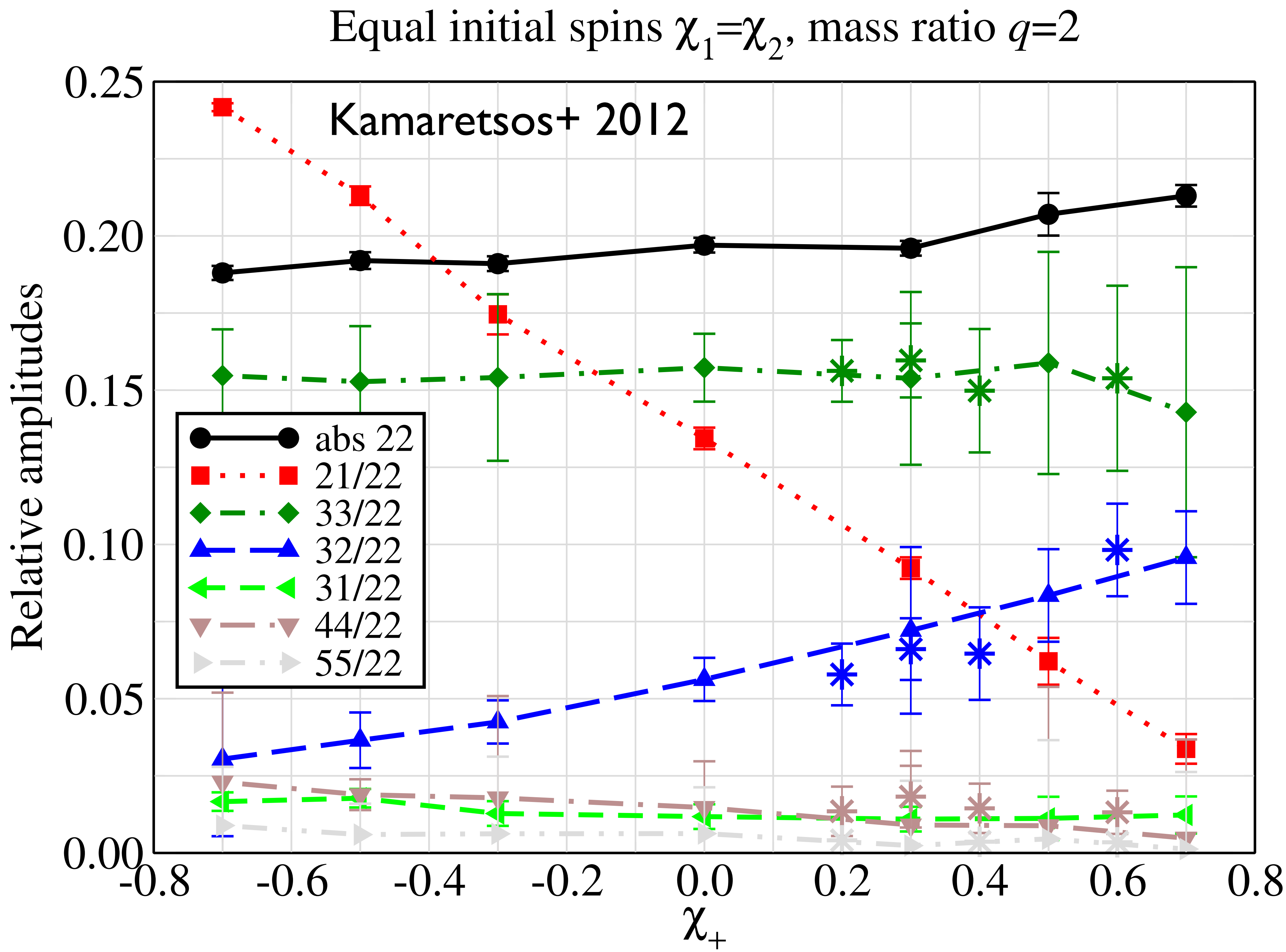
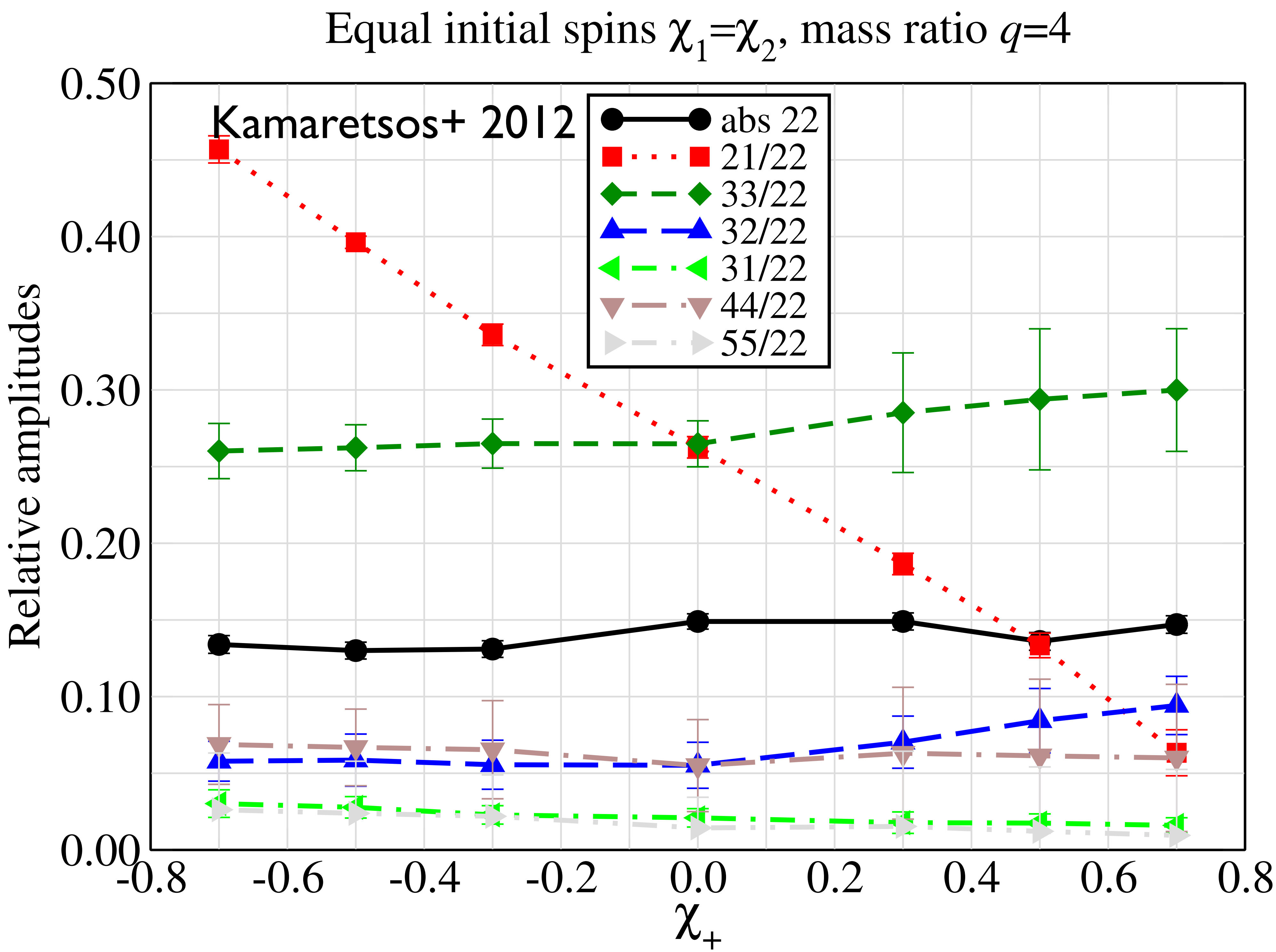
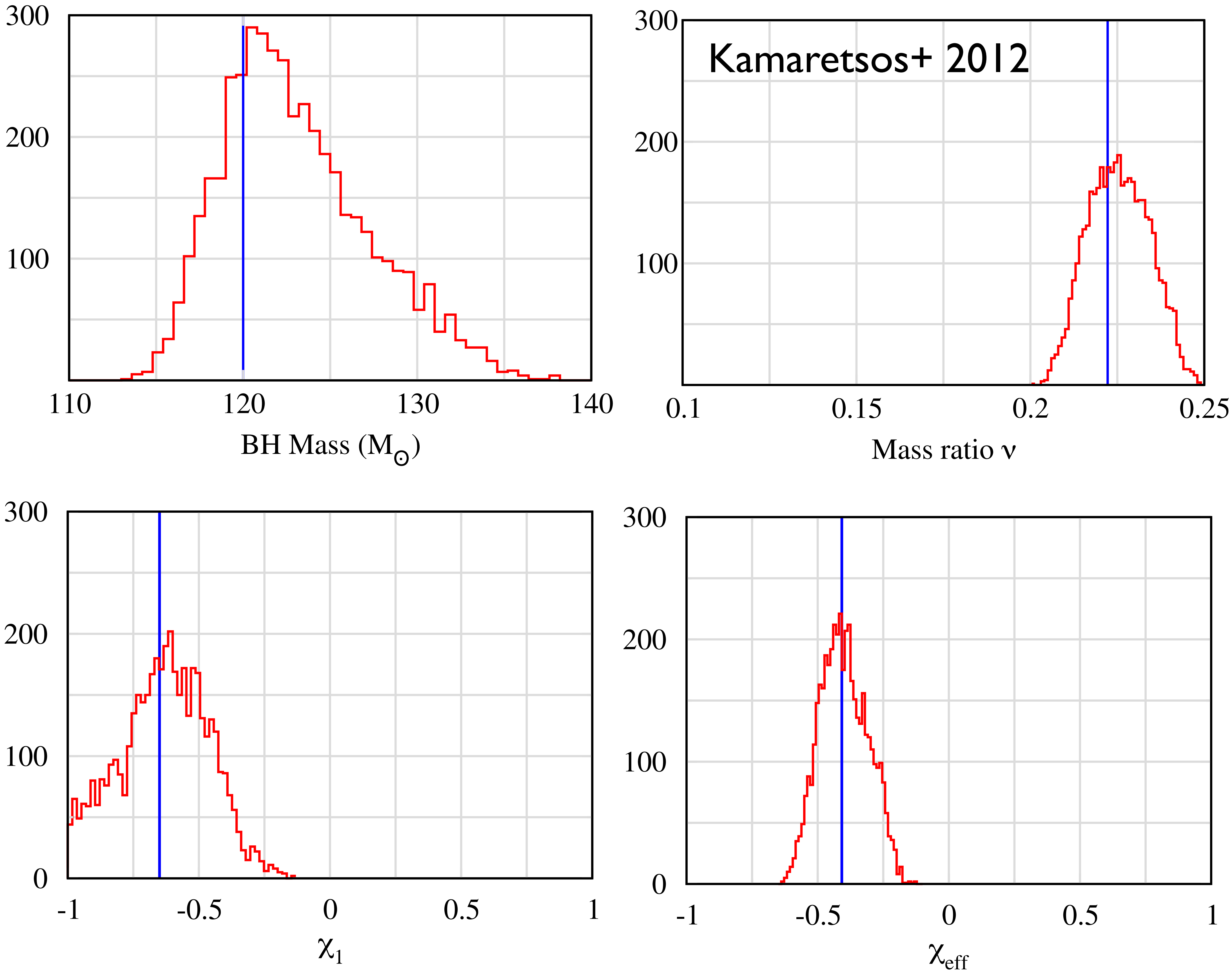


IS THIS GENERIC?



FROM RINGDOWN BACK TO INSPIRAL



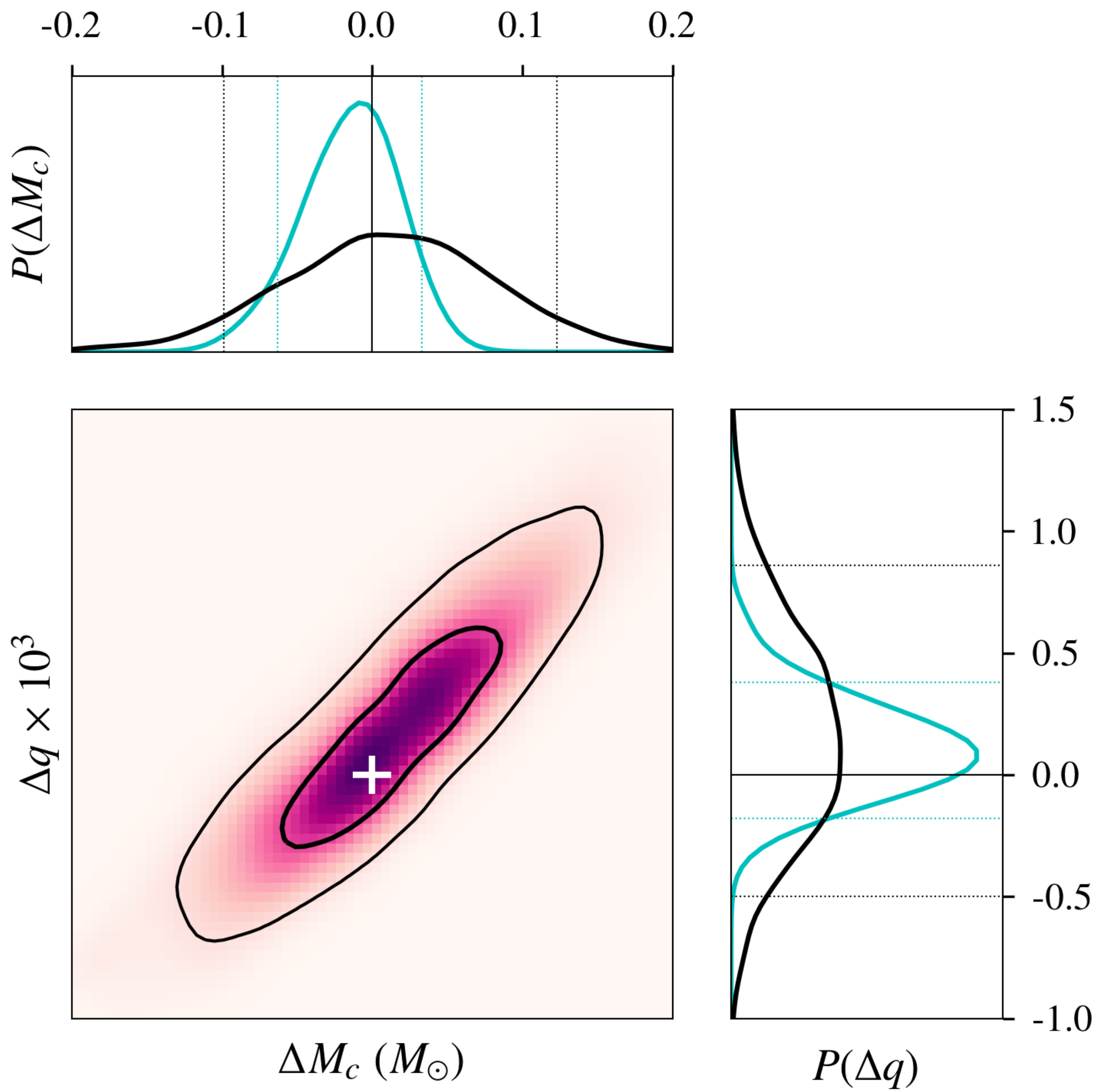
HIGHER HARMONICS IN BLACK HOLE BINARIES

$$h(t;\mathbf{n},\boldsymbol{\lambda}) = \frac{1}{d_L}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}^{-2}(\mathbf{n})\, h_{\ell m}(t;\boldsymbol{\lambda}),$$

$$\begin{aligned} h(t;\mathbf{n},\boldsymbol{\lambda},\Delta\boldsymbol{\lambda}) &= \sum_{m=\pm 2} Y_{2m}^{-2}(\mathbf{n}) h_{2m}(t,\boldsymbol{\lambda}) \\ &+ \sum_{\text{H.O.M}} Y_{\ell m}^{-2}(\mathbf{n}) h_{\ell m}(t,\boldsymbol{\lambda}+\Delta\boldsymbol{\lambda}) \end{aligned}$$

Dhanpal+ 2018

A "NO-HAIR" TEST FOR BINARY BLACK HOLES



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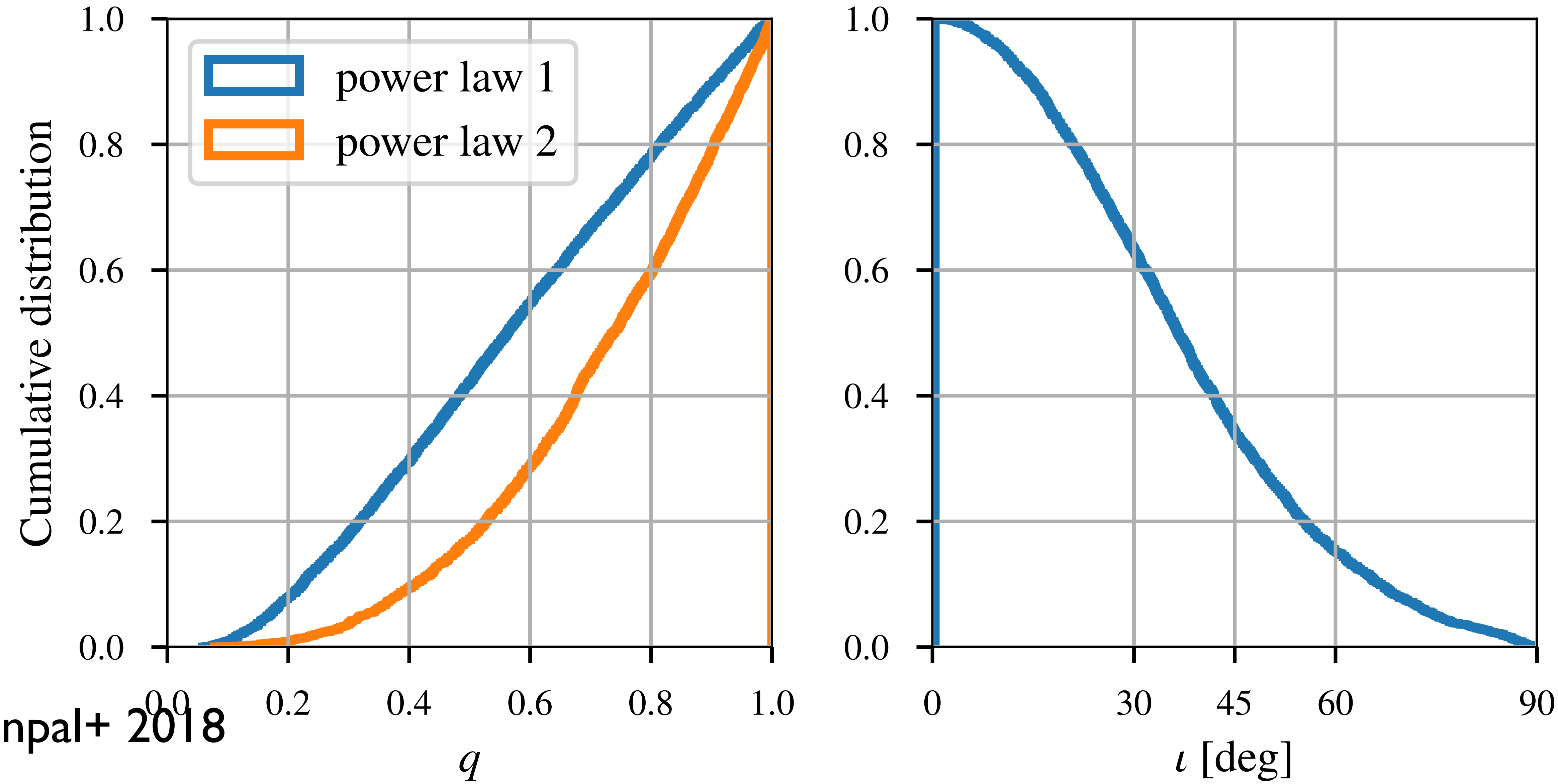
CHANCE OF DETECTING HIGHER MODES?

- test is effective only when the mass ratio is small (say < 1/2) and inclination is large (say > 60 degrees)
- only a few percent of all detected sources are suitable

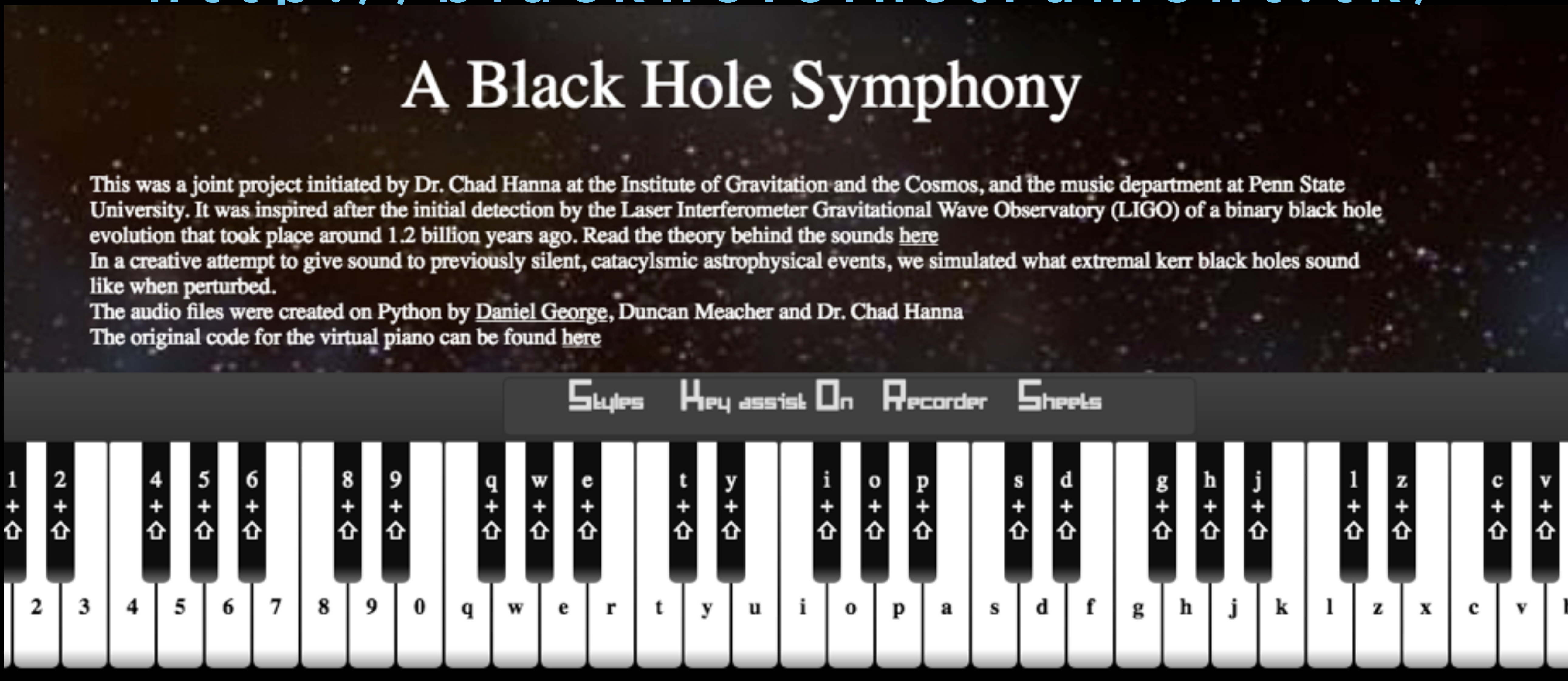


Dhanpal+ 2018

WORK DONE IN COLLABORATION WITH

- LIGO Scientific Collaboration
- Virgo Collaboration
- **Agathos**, Ajith, Arun, **Bohranian**, **Dhanpal**, Hannam, Husa, **Kamaretsos**, **Ghosh**, **Gossan**, **Meidam**, **Mehta**, van den Broeck, Veitch

LISTEN TO BLACK HOLE SYMPHONY AT: http://blackholeinstrument.tk/



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Penn State Univ.