The next theoretical challenges in gravitational-wave observations

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The science from GW experiments stems on our ability to make precise theoretical predictions of gravitational waveforms. Success of analytical/numerical-relativity interface program.

For upcoming runs, do we need more accurate or complete waveform models to interpret observations & extract information? Requirements depend on goals.

Science: observing & inferring information of NSBHs, intermediate mass BBHs; discriminating among binary formation scenarios; probing GR, EOS of NS, and cosmology:

- higher harmonics (i.e., beyond quadrupolar radiation)
- gravitational self-force formalism (for small-(or large) mass ratio binaries)
- eccentricity
- deviations from GR
- tides
Solving two-body problem in General Relativity (including radiation)

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- **GR** is non-linear theory.

- Einstein’s field equations can be solved:
  - approximately, but **analytically** (fast way)
  - exactly, but **numerically** on supercomputers (slow way)

- **Synergy** between **analytical** and **numerical relativity** is crucial.

- Physical (EOBNR) and phenomenological (Phenom) **inspiral-merger-ringdown waveforms**.
Solving two-body problem in General Relativity (including radiation)

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

• **GW151226**: SNR=13, 55 cycles (from 35 Hz), 1 sec.

(Abbott et al. PRL 116 (2016) 241103)

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- **GW170817**: SNR=32, 3000 cycles (from 30 Hz), one minute. 
  last 0.07sec

  - last minutes modeled by AR
  - modeled by NR

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(Abbott et al. PRL 119 (2017) 161101)
Unveiling binary black-hole properties: masses

Chirp mass is best measured. Individual masses can be better measured if merger is observed, because total mass is measured at merger. Spin/mass degeneracy, it can be broken by precession and higher harmonics.
Unveiling binary black-hole properties: spins

\[ \chi_{\text{eff}} = \frac{c}{GM} \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot \frac{L}{|L|} \]

- Spins along orbital angular momentum better measured.
- Spins magnitudes < 0.7.
Unveiling binary black-hole properties: spins

\[ \chi_p = \frac{c}{B_1 G m_1^2} \max (B_1 S_{1\perp}, B_2 S_{2\perp}) \]

• **Spins along orbital angular momentum** better measured.
• **Spins magnitudes** < 0.7.
• **No information** about precession.
Spin precessing waveform models

Precessing (co-rotating) frame
(AB, Chen & Vallisneri 03, Boyle et al. 11, Schmidt et al. 11, O’Shaughheessy et al. 11 )

• **Single effective-spin precessing** waveform model in **frequency domain** (IMR phenomenological, 13-independent parameters; (2,2) mode ). *(Schmidt et al. 12, Hannam et al. 14)*

• **Double-spin precessing** waveform model in **time domain** (EOBNR, 15-independent parameters; (2,2) & (2,1) modes). *(Pan et al. 14, Babak et al. 16)*
Effect of orientation of binary's orbital plane

spin nonprecessing binary

\( \chi_1 = (0,0,0.9), \chi_2 = (0,0,0.9) \quad q = 4 \)

face-on

\( \chi_1 = (0,0,0.9), \chi_2 = (0,0,0.9) \quad q = 4 \)

edge-on
Effect of orientation of binary's orbital plane

spin precessing binary

\( \chi_1 = (0.3, 0, 0.5), \chi_2 = (0.3, 0, -0.5) \quad q = 4 \)

face-on

edge-on
Binaries’ distance and inclination angle

\[ h(t) = \frac{2\mathcal{M}}{D} A(\theta, \phi, \psi; \Theta) [\mathcal{M}\omega(t)]^{2/3} \cos(2\Phi(t) + 2\Phi_0 - \alpha) \]

- So far, BBHs observed closer to face-on/face-off: are observations biased?

GW150914

GW151226

\( (Abbott \text{ et al. PRX 6 (2016) 041015}) \)
How waveform models compare with NR for BBHs observed?

(Abbott et al. PRL 116 (2016) 241103)

• Around region of original parameter recovery of BBHs, systematics due to modeling are smaller than statistical errors.
On systematics due to modeling comparing to NR waveform

- **Mock signal** from NR simulation with parameters close to GW150914.  

  \[(Abbott \ et \ al. \ CQG \ 34 \ (2017) \ 104002)\]

- **Overall, no evidence for systematic bias** relative to the statistical error of original parameter recovery of GW150914.

\[ \]
Parameter biases are found to occur for some configurations disfavored by data of GW150914.

E.g., biases are present for binaries inclined edge-on to the detector over a small range of choices of polarization angles.

Biases can be present for binaries with eccentricity $> 0.05$. 

(see also Williamson et al. 2017)
Comparing EOBNR & IMRPhenom models: detection

- **Aligned/anti-aligned** waveform models. Only dominant (2,2) mode.

[Note that only 2.1% of 100,000 points have matches < 97%.]
Comparing EOBNR & IMRPhenom models: inferring parameters

- **Aligned/anti-aligned** waveform models. Only dominant (2,2) mode.
- Differences for **large mass ratios** (> 4) and **large spins** (> 0.8).

**Faithfulness** (Bohe’, …, AB et al. 16)

[Note that only 7% of 200,000 points have matches < 97%.]
Extending waveform model in all BBH parameter space

- Difficult to run NR simulations for large mass ratios (> 4) and large spins (> 0.8), with large number of GW cycles (> 50).
  
  NR waveform with only 15 GW cycles, it constrains EOBNR model only for masses larger than 150 Msun.

- For large mass ratios (> 4) combine PN & GSF results in EOB framework.
  (Damour 09; Barausse et al. 12, Le Tiec et al. 12, Bini et al. 12-16, Antonelli et al. in progress)

- Inclusion of GSF also important for EMRIs (LISA) and IMRIs (3G detectors).
Solution? Waveforms combining NR codes

- “Best” use of finite-difference (Einstein Toolkit, ET) & pseudo spectral (SpEC) codes

\( q = 4 \quad \chi_1 = 0.9 \quad \chi_2 = 0.9 \)

(Hinder, Ossokine et al. in prep 18)
Are we missing GWs from spin precessing BBHs?

- Modeled searches in O1 & O2 used templates with aligned/anti-aligned spins.

(Apostolatos et al. 1996, AB et al. 03; Harry, Privitera, Bohe’ & AB 16)
Should we employ spin precessing searches for NSBHs?

(Harry, Privitera, Bohe’ & AB 16)

- **Spin-precessing** template bank constructed.
- Factor of about 10 increase wrt non-precessing template bank.

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- Mergers with **misaligned spins** provide unique **astrophysical insights** into **formation** scenarios.
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<td>[1, 3] $M_\odot$</td>
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<tr>
<td>$</td>
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</tr>
<tr>
<td>$</td>
<td>\chi_2</td>
</tr>
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**Diagram: 3D Illustration of Spin Alignment**

- `face-on` alignment
- `edge-on` alignment
- `face-off` alignment

**Heatmap: Sensitivity Map**

- Color scale from 1.02 to 1.50
- Sensitivity at 100 Hz
Importance of higher harmonics: varying mass ratio

\[ h_+(t; \Theta, \varphi) - i \ h_\times(t; \Theta, \varphi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} -2 Y_{\ell m}(\Theta, \varphi) \ h_{\ell m}(t) \]

(Cotesta, AB et al. '18)

*Merger-ringdown EOBNR model reproduces time & phase shifts between NR modes’ at peak.*
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- Merger-ringdown EOBNR model reproduces time & phase shifts between NR modes’ at peak.
Importance of higher harmonics also depends on geometric factor

\[ h_+(t; \Theta, \varphi) - i \ h_-(t; \Theta, \varphi) = \sum_{\ell=2}^{\infty} \sum_{m=\ell}^{-\ell} -2Y_{\ell m}(\Theta, \varphi) \ h_{\ell m}(t) \]

(Cotesta, AB et al. 18)

geometric factor is important to determine strength of higher harmonics
Accuracy of multipolar EOBNR model against NR

- Non-precessing spin EOBNR waveform model with (2,1), (3,3), (4,4) & (5,5) modes.

(Cotesta, AB et al. 18)

- Extending analysis to other modes requires producing accurate NR waveforms for those modes.

(for modeling see also Mehta et al. 17, London et al. 17; for searches see Capano, ..., AB 16, Harry et al. 18)
Intermediate-mass binary black holes (IMBBHs): search in O1

(Abbott et al. PRD96 (2017) 022001)

- **IMBHBs**: $M > 100 \text{ Msun}$, $1 < q < 10$
- Only dominant (2,2) mode used.

- **Two searches employed**: matched-filter & minimal-assumption algorithms.
- **BBH** (100 Msun, aligned spins) rate $< 0.93/(\text{Gpc})^3/\text{yr}$ at 90% confidence.
Relevance of higher harmonics for IMBBHs

- Non-spinning EOBNR waveform model with \((2,1), (3,3), (4,4)\) & \((5,5)\) modes.
  - (Pan, AB et al. 11)

- Improvements in measurement of masses & orientation angles with higher harmonics.

- Total mass better measured than chirp mass for IMBBHs.

(see also Haster et al. 15)

\[ d_L \text{ for } \text{SNR} = 12 \text{ (Gpc)} \]

\[ z \text{ Redshift} \]

\[ M_{\text{obs}} \text{ (M}_\odot\text{)} \]
Relevance of higher harmonics for IMBBHs (contd.)

(Graff, AB & Sathyaprakash 14)
Relevance of higher harmonics for IMBBHs (contd.)

SNR = 12, $M_{\text{obs}} = 500M_\odot$, $q = 4$, $\theta_{\text{JN}} = \pi/3$

dashed curves only include (2,2) mode
Unveiling the quasi-normal modes’ ringing of BHs

- **BH spectroscopy: unveiling nature of merger’s remnant**

  (Brito, AB & Raymond 18)

- We employ **parametrized inspiral-merger-ringdown** waveform model (pEOBNR) that includes modes beyond the dominant (2,2).

- Using pEOBNR we recover **more stringent bounds** on frequency and decay time of GW150914 QNM, than using damped sinusoid model.

\[
q = 6
\]
• Let us assume we did not find deviations from GR.
• We bound quasi-normal mode frequencies & decay times by combining several BH observations. \[ \sigma_{lm} = \sigma_{lm}^{GR} (1 + \delta \sigma_{lm}) \]

About 30 GW150914-like events are needed to achieve errors of 5% and test no-hair conjecture.
Eccentric waveform models

• **EOB dynamics & waveform** extended to *any* eccentricity value for nonspinning binaries.

• Binary's **degrees of freedom** are divided into a **set of phase variables**, and a **set of quantities that are constant** in the absence of radiation reaction.

\[(Hinderer \& Babak 17)\]

(for eccentricity modeling see also Huerta et al. 14, 16; Hinder et al. 17; Loutrel \& Yunes 16, 17)
How waveform models compare with NR for BNS observations?

- We rely on AR models calibrated to point-mass NR (i.e., BBHs) for long inspiral, and can tune them over last 15-20 GW cycles to NR waveforms.

(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer et al. 16, Steinhoff et al. 16, Dietrich et al. 17, Dietrich et al. 18)
A first glimpse in EOS of neutron stars: GW170817

\[ \Lambda = \frac{\lambda}{m_{NS}^5} = \frac{2}{3} k_2 \left( \frac{R_{NS} c^2}{G m_{NS}} \right)^5 \]

- With state-of-art waveform models, tides are reduced by ~20%. LVC analyses are ongoing.
- For NS spins > 0.1-0.15, spin-quadrupole affects GW phasing.

(Harry & Hinderer 17, Dietrich et al. 18)
Including dynamical tidal effects in EOB model

- **Dynamical tides**: NS’s f-modes can be excited.

- **Tidal forcing frequency approaches** eigenfrequency of NS’s normal modes of oscillation, resulting in an enhanced, more complex tidal response.

(Kokkotas et al. 1995, Flanagan et al. 08, Hinderer, … AB et al. 16, Steinhoff, … AB et al. 16)

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\[
k_2 = \frac{3}{2} \frac{\lambda}{R_{NS}^5}
\]

NS’s effective response to dynamical tidal effects

- NSBH mass ratio 2
- \( \Gamma = 2 \) polytropic
- \( C_{NS} = 0.14444 \)

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The new era of precision gravitational-wave (astro)physics

• Theoretical groundwork in **analytical and numerical relativity** has allowed us to build **faithful waveform models** to **search** for signals, **infer** astrophysical and cosmological properties and **test GR**.

• **To take full advantage of discovery potential** in next years and decades **we need** to continue to make **precise theoretical predictions**.

• For next few years, crucial (urgent) to **improve accuracy** of waveform models for mass ratios > 4 & spins > 0.8, include **higher harmonics** in spin precessing waveforms. This is **important for inferring science of BBHs, NSBH & BNS**.
“Astrophysical & Cosmological Relativity” Department

• Current members

• Past members contributed to work presented